Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 6, pp. 1535-1557

DOI: 10.4208/aamm.OA-2020-0282
December 2021

A Symplectic Conservative Perturbation Series Expansion Method for Linear Hamiltonian Systems with Perturbations and Its Applications

Zhiping Qiu* and Nan Jiang

Institute of Solid Mechanics, School of Aeronautic Science and Engineering, Beihang University, Beijing 100083, China

Received 1 September 2020; Accepted (in revised version) 24 December 2020

Abstract. In this paper, a novel symplectic conservative perturbation series expansion method is proposed to investigate the dynamic response of linear Hamiltonian systems accounting for perturbations, which mainly originate from parameters dispersions and measurement errors. Taking the perturbations into account, the perturbed system is regarded as a modification of the nominal system. By combining the perturbation series expansion method with the deterministic linear Hamiltonian system, the solution to the perturbed system is expressed in the form of asymptotic series by introducing a small parameter and a series of Hamiltonian canonical equations to predict the dynamic response are derived. Eventually, the response of the perturbed system can be obtained successfully by solving these Hamiltonian canonical equations using the symplectic difference schemes. The symplectic conservation of the proposed method is demonstrated mathematically indicating that the proposed method can preserve the characteristic property of the system. The performance of the proposed method is evaluated by three examples compared with the Runge-Kutta algorithm. Numerical examples illustrate the superiority of the proposed method in accuracy and stability, especially symplectic conservation for solving linear Hamiltonian systems with perturbations and the applicability in structural dynamic response estimation.

AMS subject classifications: 74H15, 37N15, 65P10, 65M06

Key words: Linear Hamiltonian system, perturbation series expansion method, symplectic structure, symplectic algorithm, structural dynamic response.

1 Introduction

With the dramatically rapid development of science and technology, numerical calculation has aroused more and more attention, thus there is an urgent need for more efficient

*Corresponding author.
Email: zpqiu@buaa.edu.cn (Z. Qiu)

and more stable numerical algorithms with more powerful long-term simulation capabilities.

From the perspective of mechanical systems, the conservative Newton's equation can be represented to two equivalent mathematical forms: a Lagrangian variation form and a Hamiltonian form. Compared to other forms, the Hamiltonian formulation have a symmetric form, and any physical process with negligible dissipations can be expressed in the form of a Hamiltonian system [1]. The equivalent representations describe the same physics law but provide different techniques in solving the same problem, thus may produce different numerical results [2]. Therefore, making reasonable and sensible choices from various equivalent representations are crucial.

Numerical algorithms should preserve the intrinsic properties of the original problems as much as possible [3–5]. It is worth noting that apart from some very rare exceptions, almost all the conventional algorithms are non-symplectic [6], which may lead to serious distortions of numerical results. They can be used in short-term simulation, but may result in wrong conclusions for long-term tracking research [7]. On the contrary, the symplectic algorithms of Hamiltonian systems can avoid all non-symplectic pollution and conserve the symplectic structure of the system, which is the characteristic property of the Hamiltonian system [8]. Therefore, the symplectic algorithms have significant advantages in long-term numerical simulation [9, 10].

Pioneering work on symplectic algorithms is due to Feng [11], who first proposed in 1984 at the international conference on differential geometry and equations. His work represented a milestone in the development of numerical calculation [12] and attracted extensive attention from scholars at home and abroad. Subsequently, lots of researchers obtained many major results on symplectic algorithms. The judging conditions of symplectic Runge-Kutta methods was found in 1988 by Sanz-Serna [13], Lasagni [14] and Suris [15] independently. Then, Sun [16] studied symplectic partitioned Runge-Kutta methods deeply. Around 2000, Bridges [17] and Reich [18] first put forward the multi-symplectic algorithms. In recent years, the meshless symplectic algorithms [19, 20], the symplectic continuous-stage Runge-Kutta methods [21, 22], the Fourier spectral/ pseudospectral methods [23, 24], and other symplectic algorithms have been developed in succession. A large number of numerical simulations indicate that the symplectic algorithms have superiority in conservation and long-term tracking ability.

In terms of the symplectic algorithms applied in structural dynamic response analysis, Zhang [25] proposed a symplectic algorithm for the dynamic response of the Timoshenko beam. Hu [26] utilized a multi-symplectic method to analyze the dynamic response of the multi-span continuous beam. Li [27] put forward a symplectic method for the dynamic response of the harmonic oscillator and simply supported beam. Yang [28] performed the numerical simulations of the super slender Kirchhoff rod by the symplectic algorithm. Xing [29] developed two highly precise symplectic schemes for linear structural dynamic analysis. Zhang [30] applied the symplectic Runge-Kutta method in the dynamics of spacecraft relative motion. Peng [31, 32] proposed the symplectic nonsmooth dynamic method for multibody system analysis. The numerical results all illus-