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Integral Equation Method for Inverse Scattering Problem from the Far-Field Data

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Abstract. Consider the inverse scattering problem in terms of Helmholtz equation. We study a simply connected domain with oblique derivative boundary condition. In the case of constant λ, given a finite number of incident wave, the shape of the scatterer is reconstructed from the measured far-field data. We propose a Newton method which is based on the nonlinear boundary integral equation. After computing the Fréchet derivatives with respect to the unknown boundary, the nonlinear equation is transformed to its linear form, then we show the iteration scheme for the inverse problem. We conclude our paper by presenting several numerical examples for shape reconstruction to show the validity of the method we presented.

AMS subject classifications: 35R30, 65F22, 65R20, 65R32

Key words: Helmholtz equation, oblique derivative problem, nonlinear integral equation, iterative solution, numerics.

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1 Introduction

The theory of wave field scattering plays a very important role in the field of mathematical physics. For example, the research of electromagnetic wave and acoustic wave scattering has a wide application prospect in radar remote control, space remote sensing, nondestructive testing, target stealth, medical imaging and other technologies. In a broad sense, the theory of scattering studies the relationship between medium and scattering wave. Specifically, if the total field is regarded as the sum of incident field and scattering field, the forward scattering problem is to determine the far-field mode of scattering field or scattering wave by the information and differential equation of incident field. Its mathematical essence is to solve the problem of definite solution of partial differential equation in unbounded domain. The inverse scattering problem of wave field is an important kind of inverse problem. It uses the measurement information outside the medium to detect the properties of the medium, which can not be measured directly.

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A lot of work has been done on the scattering problem of Dirichlet and Neumann boundary conditions. For the existence and uniqueness of the solution to the direct problem, see [8, 9, 12]. The authors used boundary integral equation method in [8, 12](and references therein). In [1, 9, 11, 20] the inverse problem with Dirichlet and Neumann bounday conditions were considered. The diffraction of tidal waves by an island and barriers on water of constant finite depth are modelized by the two dimensional Helmholtz equation in the free space with oblique derivative boundary conditions. The research of the problem with oblique derivative boundary is still relatively few, mainly because the boundary equation contains the tangential derivative term, which makes it difficult to deal with, but the model is suitable for any wave dynamic description, it is necessary to study it. For the scattering problem with oblique derivative boundary condition, numerical methods were given by the single-layer potential and the angular potential to the positive problem of the open curve in [14–16]. In [18], the author proved the uniqueness of the solution, then used polynomial to approximate the density function and gave the method for solving the solution, but no specific numerical examples were given. In [24], the authors considered the exterior boundary value problem for the two-dimensional Helmholtz equation with generalized oblique derivative boundary condition. They established the linear sampling method (LSM) for reconstructing the boundary of the obstacle from the far-field data.

There are many numerical methods for reconstructing the shape of scatterers, such as linear sampling method, detection method, factor decomposition method, etc. Since the inverse scattering problem is a typical nonlinear ill-posed problem, Newton method and integral equation method are often used to solve it. Newton method is to use Fréchet derivatives to transform the nonlinear equation into its linear form, then use Tikhonov regularization method to deal with the ill-posed problem, see [2, 4, 6, 7, 13, 17, 19, 22]. In [6, 7] the authors applied Fréchet derivatives in Newton method to reconstruct the impedance boundary shape of the corrosion problems. In [4, 22] the authors used full linearization and just linearized one of the equations to solve the inverse problems. The integral equation method has been used widely in [3, 5].

This paper is devoted to the numerical study of the oblique inverse scattering problem with Helmholtz equation by Newton method. More explicitly, we note that the inverse problem can be reformulated as an operator equation with respect to the boundary of the obstacle. By linearizing this operator equation, we solve the inverse problem by Newton method. First, we give an initial boundary and compute the corresponding far-field pattern. Then, after considering Fréchet differentiability of the far-field operator, we calculate the Fréchet derivatives with respect to the boundary. Finally, using regularization technique, we solve the linearized operator equation to update the boundary. Several numerical examples are presented to show the feasibility of the reconstruction scheme.

The rest of this paper is organized as follows. In Section 2, we show the inverse problem with oblique derivative boundary. Then the inverse problem of reconstructing the shape is transformed into solving the nonlinear boundary integral equation. In Section 3, we consider Fréchet differentiability of the far-field operator as the theoretical foundation