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# A Patch-Based Low-Rank Minimization Approach for Speckle Noise Reduction in Ultrasound Images

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**Abstract.** Ultrasound is a low-cost, non-invasive and real-time imaging modality that has proved popular for many medical applications. Unfortunately, the acquired ultrasound images are often corrupted by speckle noise from scatterers smaller than ultrasound beam wavelength. The signal-dependent speckle noise makes visual observation difficult. In this paper, we propose a patch-based low-rank approach for reducing the speckle noise in ultrasound images. After constructing the patch group of the ultrasound images by the block-matching scheme, we establish a variational model using the weighted nuclear norm as a regularizer for the patch group. The alternating direction method of multipliers (ADMM) is applied for solving the established nonconvex model. We return all the approximate patches to their original locations and get the final restored ultrasound images. Experimental results are given to demonstrate that the proposed method outperforms some existing state-of-the-art methods in terms of visual quality and quantitative measures.

**AMS subject classifications**: 68U10, 15A29, 65K05

**Key words**: Ultrasound images, patch, speckle noise, low-rank, weighted nuclear norm minimization.

## 1 Introduction

Ultrasound images provide the clinician with low-cost, non-invasive, and real-time images of the internal structure of the body that can help them detect deadly diseases or abnormal tissues [1–4]. As a portable and fast imaging modality, ultrasound has proved

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popular for many medical applications [5–8]. However, ultrasound images usually suffer from the signal-dependent speckle noise, limiting the contrast resolution and making it generally difficult for human interpretation and diagnosis. Therefore, speckle noise reduction is an important issue in medical ultrasound image processing [9, 10].

The ultrasound images corrupted with the signal-dependent noise can be modeled as the following form:

$$g=u+\sqrt{u}\eta, \tag{1.1}$$

where $g$ is the corrupted ultrasound image, $u$ is the original image, $\eta$ is a zero-mean Gaussian noise with standard deviation $\delta$. To restore $u$ from the corrupted ultrasound image $g$ is the main task of speckle noise reduction.

Several variational methods have been proposed to reduce the speckle noise in ultrasound images in recent decades [11]. In [12], Jin and Yang studied the following total variation based model for dealing with the speckle noise in ultrasound images:

$$\min_u \int_\Omega \frac{(u-g)^2}{u} dx + \lambda \int_\Omega |\nabla u| dx, \tag{1.2}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded set, $\lambda$ is the regularization parameter for measuring the tradeoff between the fidelity term and the regularized term, and the regularization term is the total variation of $u$. They proved the existence and uniqueness of the minimizer for the variational problem and derived the existence and uniqueness of weak solutions for the associated evolution equation. In [13], Huang and Yang proposed a convex variational model to deal with the speckle noise in real ultrasound images where the data-fitting term is the generalized Kullback–Leibler distance and the regularization term is the total variation of the estimated image.

It is widely known that the classic total variation based model usually suffers from the staircase effect. Some high-order total variation, total generalized variation, and hybrid regularization based models are proposed to overcome this shortcoming and preserve edges in the restored images well [14, 15]. In [16], Wang et al. proposed a variation model for speckle noise removal in ultrasound images where the regularization term is represented by a combination of total variation and high-order total variation (HTV) and the data fidelity term is depicted by a generalized Kullback-Leibler divergence. They solved the first-order and second-order total variation based model using the alternating direction method with multipliers (ADMM). In [17], Mei et al. investigated a second-order total generalized variation regularization to solve the problem of speckle noise reduction in ultrasound images. Numerical experiments have shown that the second-order total generalized variation based model outperforms some state-of-the-art methods in terms of visual quality, peak signal-to-noise ration (PSNR) and structural similarity (SSIM) index. In [18], Abrahim and Kadah presented a combined technique using wavelet shrinkage and total variation for speckle noise removal in clinical ultrasound images. The hybrid method takes full advantage of total variation based method to denoise the low frequency subbands without losing textures, and uses the wavelet shrinkage method based on local variance information to extract textures from noise in the high frequency subbands.