

## Buckling Analysis of FG Porous Truncated Conical Shells Resting on Elastic Foundations in the Framework of the Shear Deformation Theory

Le Kha Hoa<sup>1</sup>, Bui Gia Phi<sup>2</sup>, Do Quang Chan<sup>3,4,\*</sup>  
and Dang Van Hieu<sup>5</sup>

<sup>1</sup> Military Academy of Logistics, Hanoi, Vietnam

<sup>2</sup> Department of Technical Fundamental, University of Transport Technology, 54 Trieu Khuc, 9 Thanh Xuan, Hanoi, Vietnam

<sup>3</sup> Department of Mechanical Engineering and Mechatronics, PHENIKAA University, Hanoi 12116, Vietnam

<sup>4</sup> PHENIKAA Research and Technology Institute (PRATI), A&A Green Phoenix Group JSC, No. 167 Hoang Ngan, Trung Hoa, Cau Giay, Hanoi 11313, Vietnam

<sup>5</sup> Thainguyen University of Technology, Thainguyen, Vietnam

Received 4 July 2020; Accepted (in revised version) 14 October 2020

---

**Abstract.** In this article, an analytical method is proposed to analyze of the linear buckling behavior of the FG porous truncated conical shells subjected to a uniform axial compressive load and resting on the Pasternak elastic foundation. The material properties including Young's modulus, shear modulus and density are assumed to vary in the thickness direction. Three types of FG porous distributions including symmetric porosity distribution, non-symmetric porosity and uniform porosity distribution are considered. The governing equations of the FG porous truncated conical shells are obtained by using the first-order shear deformation theory (FSDT). With the help of the Galerkin method, the expressions for critical buckling loads are obtained in closed forms. The reliability of the obtained results is verified by comparing the present solutions with the published solutions. Finally, the numerical results show the effects of shell characteristics, porosity distribution, porosity coefficient, and elastic foundation on the critical buckling load.

**AMS subject classifications:** 74K25, 74G60

**Key words:** Buckling, truncated conical shells, first order shear deformation theory, porous materials, elastic foundations.

---

\*Corresponding author.

Emails: lekhahoa@gmail.com (L. Hoa), phibg@utt.edu.vn (B. Phi), chan.doquang@phenikaa-uni.edu.vn (D. Chan), hieudv@tnut.edu.vn (D. Hieu)

## 1 Introduction

Structures fabricated from functionally graded materials (FGMs) are usually found in kinds of engineering applications, such as pressure vessels, nuclear reactors, spacecraft, submarines, jet nozzles and other such civil, mechanical, aerospace engineering structures and chemical. The investigations on the stability, buckling, and free vibration analysis of FGM evaluated by different authors worldwide in the past few decades. Javaheri and Eslami [1] studied thermal buckling behavior of functionally graded (FG) plates by using the higher-order shear deformation plate theory. Based on Love–Kirchhoff hypothesis and Sander’s non-linear strain-displacement relation, Najafizadeh and Eslami [2] investigated the buckling analysis of FG circular plate under a radial load. Using FSDT, Liew et al. [3] analyzed thermal buckling and post-buckling of thick laminated rectangular FG plates. Based on the classical shell theory with von Karman–Donnell-type of kinematic nonlinearity, Shen [4, 5] studied the thermal post-buckling behavior of FG cylindrical shell subjected to compressive axial loads and external pressure. In [6], Shen investigated the thermal post-buckling response of a shear deformable FG cylindrical shell embedded in an elastic medium. Using the Ritz energy method and the nonlinear strain–displacement relations of large deformation, Huang and Han [7] studied the nonlinear buckling and post-buckling responses of FG cylindrical shells under axially compressive load. Duc [8] investigated the nonlinear thermal dynamic behavior of imperfect FG cylindrical shells reinforced by outside stiffeners and surrounded on elastic foundations using the third-order shear deformation shells theory in thermal environment. The nonlinear response of thick sigmoid FG cylinder with temperature-independent material property surrounded on elastic layers and subjected to mechanical and thermal loads was investigated by Duc et al. [9]. Using both of the first-order shear deformation theory and stress function with full-motion equations, Duc and Thang [10] studied the nonlinear dynamic response and vibration of imperfect ES-FGM cylinder subjected to mechanical and damping loads and surrounded on elastic layers. Dung and Hoa [11] investigated the thermal nonlinear buckling and post-buckling of FG stiffened cylindrical shells embedded in a Pasternak medium and subjected to torsional load. Based on the Donnell theory of shells and the von-Karman type of geometrical nonlinearity, Sabzikar Boroujerdy et al. [12] investigated thermal buckling behavior of cylindrical shells resting on the Pasternak foundation with regard to temperature dependency of the constituents. Using the generalized differential quadrature method and FSDT, Tornabene et al. [13] investigated the static and dynamic of laminated doubly-curved shells and panels of revolution resting on the elastic foundation. Duc and Quan [14] investigated the nonlinear dynamic response and vibration of imperfect ES-FGM double curved thin shallow shells on elastic foundation. In [15], based on Reddy’s higher-order shear deformation shell theory, Duc and Quan analyzed the nonlinear static and dynamic stability of imperfect eccentrically stiffened FG double curved shallow shell on elastic foundations in thermal environments. Sofiyev and Aksogan [16] studied the buckling of a truncated conical shell under a uniform external pressure. The shell is assumed to have a meridional thickness described