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Recent Advances and Emerging Applications of the Singular Boundary Method for Large-Scale and High-Frequency Computational Acoustics

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Abstract. With the rapid development of computer technology, numerical simulation has become the third scientific research tool besides theoretical analysis and experimental research. As the core of numerical simulation, constructing efficient, accurate and stable numerical methods to simulate complex scientific and engineering problems has become a key issue in computational mechanics. The article outlines the application of singular boundary method to the large-scale and high-frequency acoustic problems. In practical application, the key issue is to construct efficient and accurate numerical methodology to calculate the large-scale and high-frequency sound field. This article focuses on the following two research areas. They are how to discretize partial differential equations into more appropriate linear equations, and how to solve linear equations more efficiently. The bottle neck problems encountered in the computational acoustics are used as the technical routes, i.e., efficient solution of dense linear system composed of ill-conditioned matrix and stable simulation of wave propagation at low sampling frequencies. The article reviews recent advances in emerging applications of the singular boundary method for computational acoustics. This collection can provide a reference for simulating other more complex wave propagation.

AMS subject classifications: 65N80, 65N35, 65N38, 86-08

Key words: Singular boundary method, origin intensity factor, high-frequency acoustic problems, large-scale acoustic problems, Helmholtz equation.

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1 Introduction

Large-scale and high-frequency sound field calculations play an important role in scientific calculations and engineering analysis, such as the submarine underwater noise detection [1] and underwater sonar imaging detection [2]. Currently, there are two technical means to analyse the propagation of high-frequency waves, i.e., high-frequency analysis and numerical simulation. High-frequency analysis is widely applied in simulation of high-frequency wave propagation due to less storage requirements and less amount of calculation, such as the geometrical theory of diffraction [3], the physical theory of diffraction [4] and the geometric optics method [5]. However, the high-frequency analysis has drawbacks of lower accuracy and lower stability. In addition, the high-frequency methods would lose stability in low-frequency situation. Compared to high-frequency analysis, numerical simulation can simulate wave propagation in a wide frequency range and obtain highly accurate solutions, such as the boundary element method (BEM) [6,7], the finite element method (FEM) [8,9], the artificial neural network methods [10,11] and the singular boundary method (SBM) [12–14], etc. However, numerical simulation requires powerful computational facilities. Therefore, traditional numerical methodologies can only analyse the propagation of low-frequency and intermediate-frequency waves.

Numerical simulation has been considered as the third scientific research tool besides theoretical analysis and experimental research. Essentially, numerical simulation is a computer-based technology for solving partial differential equations (PDE) as depicted in Fig. 1. Generally speaking, the first step in dealing with a physical problem is to construct the corresponding PDE to describe physical phenomena, such as the convection-diffusion-reaction equations [15,16] and the Maxwell's equations [17]. In step 1, some simplified conditions are inevitably added to construct a mathematical model. Therefore, model errors are generated. Secondly, the PDE is discretized into linear equations by corresponding numerical methods, such as the FEM and the BEM. In step 2, the mathematical model with infinite degrees of freedom (DOF) is discretized into linear equations with finite DOF. Therefore, the discretization error is generated inevitably. Finally, the linear equations are solved by appropriate numerical solver, such as the Gauss solver and the generalized minimal residual algorithm (GMRES) [18]. Step 3 inevitably generates truncation errors due to limitation of the effective storage digits of computer. Model error, discretization error and truncation error constitute the three main sources of error in numerical simulation. How to efficiently reduce these errors and balance their influence on results directly determines the effectiveness and accuracy of numerical simulations.

This paper provides a collection of newly emerging numerical techniques for large-scale and high-frequency acoustic problems. This work mainly focus on how to deal with the above steps 2 and 3 in the numerical simulation more efficiently, i.e.,

(1) How to discretize PDE into more appropriate linear equations;

(2) How to solve linear equations more efficiently.