Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **14**, No. 2, pp. 386-414

DOI: 10.4208/aamm.OA-2020-0327
April 2022

A Modified Characteristics Finite Element Method for the Electroneutral Micro-Fluids

Zhiyong Si*, Piaopiao Dong and Yunxia Wang

School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, Henan 454003, China

Received 25 October 2020; Accepted (in revised version) 28 March 2021

Abstract. In this paper, a numerical method for the electroneutral micro-fluids based on the finite element method will be given. In order to deal with the non-linearity of the equation, the modified characteristics method will be used to deal with the temporal derivates term and the convective term. In this way, the non-linear equation can be linearlized. Then, we will give the unconditional stability and optimal error estimation. At last, some numerical results are given to show the effectiveness of our method. From the stability analysis we can see that the method is unconditionally stable. The numerical results show that our method is robust.

AMS subject classifications: 76M10, 65N12, 65N30, 35K61

Key words: Electroneutral micro-fluids, finite element method, modified characteristics method, unconditional stability, optimal error estimation.

1 Introduction

In the past decades, the micro-fluids have been widely studied, which is widely used in the modern biological, chemical and medical analysis technology, used to replace conventional experimental operations, and has brought new revolutionary capabilities to biology, chemistry and medicine. The flows in the macroscopic scale and microsystems are not quite the same as the flows in the normal scale. There is great need to develop numerical methods for the micro-fluids, and there are many great works in this area. Bianchi et al. gave a finite element formulation for the simulation of an electroosmotic flow in rectangular micro-scale channel networks [7]. In [15], Jang and Lee gave a micro-pump of which pumping mechanism is based upon MHD principles. In [16], Li and Peterson gave a full 3-dimensional (3D) conjugate heat transfer model to simulate the heat transfer performance of siliconbased, parallel microchannel heat sinks. In [20],

*Corresponding author.
Emails: sizhiyong@hpu.edu.cn (Z. Si), 1138436589@qq.com (P. Dong), wangyunxia@hpu.edu.cn (Y. Wang)

B. Mohammadi and J. Tuomela presented a micro-fluidic flow model where the movement of several charged species is coupled with electric field and the motion of ambient fluid. A numerical study on laminar forced convection of water in offset strip-fin microchannels network heat sinks for microelectronic cooling was presented by Hong and Cheng [13]. In [19], Mohammadi and Santiago showed a numerical method which combines the combination of a state control and shape design approaches for the optimization of micro-fluidic channels used for sample extraction and separation of chemical species existing in a buffer solution. An adaptive finite element method with large aspect ratio for mass transport in electroosmosis and pressure-driven microflows was shown by Prachittham et al. [24]. Singh and Agrawal [29] presented the Burnett equation in cylindrical coordinates and the solution in a micro-tube. Xia et al. [34] studied the effect of geometric parameters on water flow and heat transfer characteristics in microchannel heat sink with triangular reentrant cavities using numerical method. Huang et al. [14] studied the heat transfer characteristics of a double-layered microchannel heat sink by finite volume method. Sui et al. [31] studied the fully-developed flow and heat transfer in periodic wavy channels with rectangular cross sections by the FLUENT software. In [12], Ho and Hung presented hybrid finite element and particle-in-cell simulation of the effect of different Debye lengths on charged ion migration in capillary zone electrophoresis. In [8], Davydova et al. established the best possible design for a flow distributor at the entrance of a flat channel. Lockerby and Collyer [18] derived the fundamental solutions (Green's functions) to Grad's steady-state linearised 13-moment equations for non-equilibrium gas flows. Xia et al. [35] studied micro-piv visualization and numerical simulation of flow and heat transfer in three micro pin-fin heat sinks. Yang et al. [36] analyzed the measurement performance of ions in electrophoresis microchips with different crosses comparatively by simulation and experimental methods. In [22], Pezeshkpour gave a shape factor model for injection analysis of microchip sample electrophoresis. Abdollahi et al. [1] studied the fluid flow and heat transfer of liquid-liquid taylor flow in square microchannels. Li et al. [17] gave the numerical approach for nanofluid transportation due to electric force in a porous enclosure. Wang [32] presented the numerical investigation on the heat transfer of a droplet-laden flow in a Microfluidics system based on the volume of fluid method. In [33], a new error analysis of characteristics-mixed fems for miscible displacement in porous media was given.

The method of characteristics is effective for non-linear convection problem, which can linearlize the non-linear term. Numerical analysis of the method has been done extensively. In this method, the hyperbolic part (the temporal and advection term) is approximated usually by a characteristic tracking scheme, such as

$$(u_t+b\cdot\nabla u)|_{t=t^{n+1}}\approx\frac{u^{n+1}(x)-u^n(x-b^n\tau)}{\tau},$$

which may provide a more accurate approximation in the characteristic direction. The modified characteristics method with both finite difference and finite element approximations for linear convection-dominated diffusion problems was repsented by Douglas