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Alternating Cell Direction Implicit Method using Approximate Factorization on Hybrid Grids

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Abstract. In this study, a novel fast-implicit iteration scheme called the alternating cell direction implicit (ACDI) method is combined with the approximate factorization scheme. This application aims to offer a mathematically well-defined version of the ACDI method and to increase the accuracy of the iteration scheme used for the numerical solutions of partial differential equations. The ACDI method is a fast-implicit method that can be used for unstructured grids. The use of fast implicit iteration methods with unstructured grids is not common in the literature. The new ACDI method has been applied to the unsteady diffusion equation to determine its convergence and time-dependent solution ability and character. The numerical tests are conducted for different grid types, such as structured, unstructured quadrilateral, and hybrid polygonal grids. Second, the ACDI was applied to the unsteady advection-diffusion equation to understand the time-dependent and progression capabilities of the presented method. Third, a full potential equation solution is created to understand the complex flow solving ability of the presented method. The results of the numerical study are compared with other fast implicit methods, such as the point Gauss–Seidel (PGS) and line Gauss–Seidel (LGS) methods and the fourth-order Runge-Kutta (RK4) method, which is an explicit scheme, and the Laasonen method, which is a fully implicit scheme. The study increased the abilities of the ACDI method. Due to the new ACDI method, the approximate factorization method, which is used only in structural grids that are known to be advantageous, can be applied to any mesh structure.

AMS subject classifications: 68Q01, 65Y04, 65Y20, 35D99, 74G15

Key words: Approximate factorization, ACDI method, fast implicit schemes.

1 Introduction

The numerical solution to partial differential equations is the basis of scientific computation applications. Solutions are found using discrete forms of equations with finite

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instead of infinitesimal cells. The discretization approaches of equations specify the classification of the solution methods. One of the classifications is having an implicit or explicit formulation. Implicit formulations result in linear sets of equations in matrix form, whereas explicit formulations directly use the previous iteration or time-step values to obtain the current iteration or time-step value. Explicit schemes are more common for unstructured grids [1,2].

Implicit formulations are approaches that are more difficult to program than explicit schemes and have better convergence. Being more convergent can be an advantage if fast implicit methods, such as the alternating direction implicit (ADI) method, are applied. These kinds of fast implicit formulations are usually appropriate for structured grids [1,2] that are composed of ordered finite cells on the solution domain. Their formulations generally use tri-diagonal or pentadiagonal matrix solutions, and these kinds of matrices are relatively easier to solve. Fully implicit schemes require more computational time because they result in mass matrices for the solution, and the efforts to obtain quicker results with these applications are generally focused on faster solution methods of mass matrices [3].

The alternating cell direction implicit (ACDI) method is a fast-implicit scheme that can be used for both structured and unstructured grids [4,5]. This ability is not common for fast implicit schemes. Dimitri [2] stated that it is possible to generate grid directions with the edges of triangular elements and that these directions can be used for line implicitness. However, these sweep directions are not unique for an unstructured grid, and they are defined by a method selected by the programmer. One of the most important studies on line implicitness of unstructured grids is on the usage of Hamiltonian Tours [6,7]. Venkatakrishnan [3] stated that many Hamiltonian Tours might exist and that there might be none for a two-dimensional (2D) unstructured mesh.

The ACDI method is offered to combine the advantages of the successful convergence of fast implicit methods with the easier meshing advantage of unstructured grids. The method is inspired by ADI methods by Meyer [8], Kellog [9], and Vries [10], but its algorithm structure is similar to that of the line Gauss–Seidel (LGS) iteration method, which is another fast-implicit method for the existence of quadrilateral elements. Because one of the objectives of this study is to increase the implicitness of the ACDI method to the order of approximate factorization [11,12], the classical approximate factorization approach is explained briefly after the point Gauss–Seidel (PGS), LGS, ACDI, and full implicit Laasonen methods.

The main aim of this study is to combine an approximate factorization scheme with the ACDI [11,12] time iteration scheme to increase the implicitness and accuracy of the previous ACDI studies. Although the main study is conducted on using cell-centered finite volume discretization, the approximate factorization approach used for this study is also explained for node-based finite difference discretization on structured grids for clearness, and it is aimed to serve as a mathematically well-defined approach. The unsteady diffusion, unsteady advection-diffusion, and transonic full potential equations are selected as the model equations for the validation of the method.