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A Cell-Centered Lagrangian Scheme with an Elastic-Perfectly Plastic Solid Riemann Solver for Wave Propagations in Solids

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Abstract. A cell-centered Lagrangian scheme is developed for the numerical simulation of wave propagations in one dimensional (1D) elastic-plastic flow. The classical elastic-plastic material model initially proposed by Wilkins is adopted. The linear elastic model (Hooke's Law), perfectly plastic model and von Mises yield criterion are used to describe the constitutive relationship of elastic-plastic solid. The second-order extension of this scheme is achieved by a linear reconstruction method. Various numerical tests are simulated to check the capability of this scheme in capturing nonlinear elastic-plastic waves. Compared with the well-developed operator splitting method used in simulating elastic-plastic flow, this scheme is more accurate due to the consideration of a list of 64 different types of the nonlinear elastic-plastic waves when constructing the elastic-perfectly plastic Riemann solver. The numerical simulations of typical examples show competitive results.

AMS subject classifications: 35Q74, 74B10, 74C05, 74M20

Key words: Elastic-plastic flow, cell-centered Lagrangian scheme, elastic-perfectly solid Riemann problem, wave propagation.

1 Introduction

The elastic-plastic flow is commonly seen in engineering when solid materials are subjected to high impulse loads. Complex elastic/plastic wave propagate accompanied with discontinuities such as jumps of density, velocity and stress [1–4]. When investigating elastic-plastic problems, experimental researches have encountered large difficulties for

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the strong nonlinear features [5, 6]. Therefore, numerical simulation has become an efficient research method to study wave structures, material interfaces, elastic-plastic phase transitions and physical discontinuities in elastic-plastic flow [7]. Staggered Lagrangian method is usually used to investigate the elastic-plastic problem, which is introduced by von Neumann and Richtmyer [8] and developed by Wilkins [9]. Lagrangian methods are characterized by the mesh moving with the fluid. The kinematical variables (velocity and momentum) are defined at nodes while the other variables (density, pressure, stress and specific internal energy) are located at the center of cells. Artificial viscosities are needed to ensure entropy production and suppress spurious numerical oscillations around the shock waves in staggered Lagrangian method. The scheme can capture interfaces in a natural manner and calculate stress in high precision. Cell-centered Lagrangian method, which is introduced by Godunov in one dimensional Lagrangian gas dynamics [10], is also used to investigate the elastic-plastic problem recently [11–14]. Different from staggered Lagrangian methods, velocity and momentum are defined at the center of the cell. In the discrete frame of finite volume methods, computational flux across cell boundary is computed by solving some kind of discontinuity problem in the direction normal to the boundary. Thus the conservative property can be ensured and it is not necessary to use artificial viscosity. In the perspective of an arbitrary Lagrangian-Eulerian extension of a Lagrangian scheme [15], the consistency among the control volumes for the advection of mass, momentum and energy is also kept. The nodal velocity is usually calculated by the velocities of surrounding boundaries obtained from the discontinuity problems. [16, 17] Therefore solvers to these discontinuity problems are vital in cell-centered Lagrangian schemes.

In the early days of cell-centered elastoplastic algorithm design, such as Caveat code [18], the operator splitting method is adopted to divide the algorithm into two main steps: a Lagrangian fluid step and an elastoplastic step. In the Lagrangian fluid step, the numerical fluxes are calculated by a fluid Riemann solver for fluid discontinuity problem. Various fluid Riemann solvers have been designed to reconstruct the numerical flux [19–21]. Fluid primitive variables (density, velocity and thermodynamic pressure) and geometric variables (position of vertex) are then updated. In the elastoplastic step, deviatoric stresses are calculated by the geometric variables. Then primitive variables are modified by deviatoric stresses again. Such splitting methods obtain good numerical simulations in some elastic-plastic problems. But when there exist complex wave structures in the discontinuity problems of solid and fluid, the elastic-plastic deformation is caused so tightly coupled by two steps that it is not suitable for splitting. Thus a direct idea is to develop a unsplit algorithm to solve an elastoplastic discontinuity problem.

For the purpose of combining the fluid Riemann problem with the elastic-plastic property, many elastic-plastic Riemann solvers based on appropriate constitutive model of solid have been developed. Lin [2] adopted the Riemann invariants of the corresponding linearized equations and developed an iterative procedure to solve the elastic-plastic Riemann problem. Cheng [13] constructed a two-rarefaction fluid waves with elastic waves (TRRSE) for a non-conservative system. Ortega [22] presented a multi-material