

# Domain-Decomposition Localized Method of Fundamental Solutions for Large-Scale Heat Conduction in Anisotropic Layered Materials

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**Abstract.** The localized method of fundamental solutions (LMFS) is a relatively new meshless boundary collocation method. In the LMFS, the global MFS approximation which is expensive to evaluate is replaced by local MFS formulation defined in a set of overlapping subdomains. The LMFS algorithm therefore converts differential equations into sparse rather than dense matrices which are much cheaper to calculate. This paper makes the first attempt to apply the LMFS, in conjunction with a domain-decomposition technique, for the numerical solution of steady-state heat conduction problems in two-dimensional (2D) anisotropic layered materials. Here, the layered material is decomposed into several subdomains along the layer-layer interfaces, and in each of the subdomains, the solution is approximated by using the LMFS expansion. On the subdomain interface, compatibility of temperatures and heat fluxes are imposed. Preliminary numerical experiments illustrate that the proposed domain-decomposition LMFS algorithm is accurate, stable and computationally efficient for the numerical solution of large-scale multi-layered materials.

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**Key words:** Meshless method, localized method of fundamental solutions, heat conduction problems, layered materials, large-scale problems.

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## 1 Introduction

The method of fundamental solutions (MFS) belongs to the family of meshfree boundary collocation methods and nowadays has been extensively applied to many kinds of

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engineering applications [1–9]. The method is simple in derivation, efficient in calculation, and won the favor of many scientists and researchers in science and engineering fields. However, as a price paid for such advantages, the traditional global-based MFS formulation will produce a dense rather than a sparse matrix which is expensive to calculate. This presents a serious problem for the method for large-scale engineering simulations [4, 10–20].

Very recently, a modified MFS formulation, named as the localized method of fundamental solutions (LMFS) [21–23], is presented to circumvent some of the aforementioned bottlenecks. In the LMFS, the entire computational domain is firstly discretized into a set of overlapping random imbricated subdomains, and in each of the subdomain, the traditional MFS formulation and the moving least square method are then applied to form the local systems of linear equations. By fulfilling boundary condition at boundary nodes and governing equations at interior nodes, the LMFS algorithm finally converts the original differential equations into sparse rather than dense matrices because only a handful of basis functions are non-zero in a given sub-domain. One of the motives for such “localizing” or “sub-structuring” is that it can convert differential equations into sparse rather than dense matrices which are much cheaper to calculate [12, 18, 24–28]. Prior to this study, the LMFS has been successfully applied for 2D and 3D Laplace equations [21, 23], 2D elasticity problems [22], 3D Helmholtz-type equations [29] and inverse problems [30].

In this paper, we document the first attempt to apply the LMFS, in conjunction with a non-overlapping domain decomposition technique [31], for the numerical solution of heat conduction in 2D anisotropic layered materials. Here, the layered material is decomposed into several subdomain along the layer-layer interfaces, and in each of the subdomain, the solution is approximated by using the LMFS expansion. On the subdomain interface, compatibility of temperatures and heat fluxes are imposed. In recent decades, the combination of the domain decomposition technique and the MFS has been proposed for the numerical solutions of many problems in engineering applications. In [32], Alves et al. used the domain decomposition methods with fundamental solutions to deal with Helmholtz problems with discontinuous source terms. In [33], Xiao et al. applied the MFS in conjunction with domain decomposition technique for the numerical solution of free surface problems in layered soil. For an overview of the state of the art, we refer interested readers to articles [31–34] for existing theoretical results, different algorithms and/or software packages.

We start in Section 2 by describing the basic mathematical theory of the LMFS for the numerical solution of 2D anisotropic heat conduction problems. In Section 3, a domain decomposition LMFS (DD-LMFS) scheme is proposed for solving multi-layered heat conduction problems. Next in Section 4, three benchmark numerical examples are studied to validate the performance of the method. Finally, some conclusions and remarks are provided in Section 5.