Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **14**, No. 4, pp. 816-841

DOI: 10.4208/aamm.OA-2020-0177
August 2022

Dynamic Loadings Induced Vibration of Third Order Shear Deformable FG-CNTRC Beams: Gram-Schmidt-Ritz Method

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Received 2 August 2020; Accepted (in revised version) 22 December 2020

Abstract. This research work deals with a study on dynamic behavior of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) beams under various types of dynamic loads. Carbon nanotubes (CNTs) are used as the reinforcing materials that distribute continuously across the beam thickness. By using third order shear deformable theory (TSDT) in this current study, the straightness and normality of the transverse normal after deformation are unconstrained. The equations of motion based on TSDT are solved by Gram-Schmidt-Ritz method in which the displacement functions are generated via Gram-Schmidt procedure. Additionally, the time-integration of Newmark is also employed to carry out dynamic response of the beams under dynamic loads. Several effects such as material distributions, types of dynamic loads, boundary conditions and so on are taken into account. According to numerical results, it can be revealed that adding small amount of CNTs can reduce considerably the dynamic amplitude of FG-CNTRC beams. Moreover, the dynamic analysis of beam-like structures plays an important role in structural design because mass inertia matrix of the beam being involved in the equations of motion, which yields much larger deflection than that predicted by simple static analysis.

AMS subject classifications: 65M10, 78A48

Key words: CNTRC beam, dynamic loads, moving load, gram-schmidt procedure, third order shear deformation theory.

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1 Introduction

One of currently promising materials is carbon nanotubes (CNTs) that exhibit exceptional physical, mechanical, and electrical properties. CNTs have been used in various applications of nanotechnology, electronics, optics and material sciences [1–3]. By using CNTs as reinforcing materials in polymeric matrix, we can receive an advanced class of composite materials which is much better than traditional composites. Recently, high performance structural composites with multiple properties are needed in engineering communities. With help of CNTs, the advanced composite materials are excellent in high stiffness and low density. For example, adding only 1.8 vol% of graphene in poly (vinyl alcohol) composites leads to a 150% enhancement in tensile strength and around 10 times increase in Young's modulus [4]. However, the critical challenge of producing polymer/CNT composites is how to enhance dispersion and alignment of CNTs in a polymer matrix. Xie et al. [5] reviewed available techniques and recent progress on dispersion and alignment of CNTs in the polymer matrix using ex situ technique, force and magnetic fields, electrospinning and liquid crystalline phase induced methods. The earliest study was found in [6] that presented carbon nanotube-reinforced composites (CNTRCs) made from polymer reinforced by aligned CNT arrays. Since then, some investigations on material properties of CNTRCs were reported in [7–9] and the numerical analysis and experiment of CNTRCs were reviewed by Yengejeh et al. [10].

An idea of functionally graded (FG) materials is applied to create new type of CNTRCs in which the volume fraction of CNTs is changed gradually across the desirable direction. Therefore, the new type of composites is called functionally graded carbon nanotube reinforced composites (FG-CNTRCs). Recent research activities and some highlight topics relevant to FG-CNTRCs were reviewed by Liew et al. [11]. For structural analysis, there exists number of investigations on static bending, buckling and vibration of FG-CNTRC structures [12–15]. Moreover, Rafiee et al. [16] showed the large amplitude vibration of FG-CNTRC beams with piezoelectric layers. Based on their results, it is revealed that natural frequency and the nonlinear to linear frequency ratio increase as the increase of CNT volume fraction. The nonlinear analysis of FG-CNTRC structures has been of interest to many researches since the past until now. They have investigated nonlinear bending, post-buckling and nonlinear vibration of FG-CNTRC beams, plates and shells [17–24] By using the incremental harmonic balance (IHB) method, the super harmonic resonances of FG-CNTRC was derived in the study of Zhihua et al. [25]. Karami et al. [26] used second order shear deformation plate theory for analyzing static, stability and vibration of FG-CNTRC plates including size-dependent parameters. With the Donnell-type kinematic assumption, Qin et al. [27] applied Chebyshev polynomials to solve the free vibration problem of rotating FG-CNTRC cylindrical shells. Natural frequencies associated with their mode shapes of circular, annular and sector plates made of FG-CNTRC were presented by Zhong et al. [28] using the Ritz variational energy method. For small-scale structures, it has been proven that the size-dependent parameters play a vital role in mechanical characteristics of such small structures [29]. Finite element