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Chebyshev-Lagrange Multipliers Technique for Vibration Analysis of Functionally Graded Material Beams using Various Beam Theories

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Abstract. In this paper, a new technique for analysing functionally graded material (FGM) beams using the Chebyshev polynomials and Lagrange multipliers with various beam theories is presented. By utilizing the inner products and the Chebyshev polynomials' orthogonality properties incorporated with Lagrange multipliers, we can combine the governing equation and boundary conditions to yield the matrix equations with explicit weighting coefficients. Numerical examples are provided for vibration analysis of various beam theories and assumptions. Based on numerical evaluations, it is revealed that the proposed technique can efficiently achieve good agreement with those of the references.

AMS subject classifications: 41A50, 65D15, 65D20, 74H45

Key words: Chebyshev polynomials, Lagrange multipliers, functionally graded material, orthogonality, vibration analysis.

1 Introduction

Functionally Graded Materials (FGMs) are usually made of metal-matrix composites and varying volume fractions of the constituent mixture through the thickness direction resulting in superior material properties. Its features, such as the high stiffness-to-weight and strength-to-weight ratio, apply to various and extensively used in spacecraft, aerospace, and advance structural engineering materials [1]. One of the most challenging related to FGM problems in engineering structures is analysing a complex response of

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FGM materials. Recently, Hu et al. [2] developed an analysis of functionally graded Timoshenko microbeams incorporated with modified couple stress theory. An extended isogeometric analysis (IGA), the non-uniform rational B-spline, so-called NURBS, is the basic function to describe FG curved beams' exact geometry. A precise geometry of a curvilinear domain is mapped continuously under the parametric domain by the adopted B-spline basis functions for an accurate displacement approximation. A superior analysis performance can be obtained with their approach. Functionally graded materials are extended and characterized for 2-D elastodynamic problems based on the Mori-Tanaka micromechanics material in conjunction with a truly meshfree radial point interpolation method (t-RPIM) by Bui et al. [3]. An improved version of a t-RPIM method is achieved by utilizing the so-called Cartesian transformation method (CTM) for domain integrations. Higher accurate results can be obtained in comparison with the conventional Gaussian quadrature (GQ) RPIM.

FGMs are inhomogeneous composite materials with gradients of a specific composition. The properties can be described through the thickness, for example, using the power-law form of Simsek [4]. This leads to a complicated formulation of FGM characteristics, especially for vibration analysis. Simsek [4] provided different higher-order beam assumptions of FGM beams with the Lagrange multipliers method to obtain fundamental frequencies. An attractive means of the Lagrange multipliers incorporated with strain energy was proposed. The strain energy and boundary conditions are entirely separated from the approximate functions. The Lagrange multipliers method can provide developments of explicit weighting coefficients. Recently, an efficient way to analyze functionally graded beams in an explicit algebraic form was proposed by Banerjee and Ananthapuvirajah [5], namely the dynamic stiffness method.

The Chebyshev polynomials are considered and chosen according to their orthogonality property to provide an efficient computation for the advanced materials. Celik [6] initially introduced the use of the Chebyshev collocation method (CCM) to analyze and compute higher Strum-Liouville eigenvalues, and the corrected eigenvalues can be obtained corresponding to this technique.

Sari and Butcher [7] used the Chebyshev polynomials to analyze natural frequencies and critical loads of beams and columns with damaged boundaries. Sari and Butcher [8] employed the Chebyshev collocation method to analyze vibrations of rotating and nonrotating beams with damage boundaries. Timoshenko's beam theory is also considered by introducing a recurrence differentiation property known as the Chebyshev differentiation matrix of the Chebyshev polynomials evaluated at the Chebyshev points. In their approach, the Chebyshev differentiation matrix is explicitly weighted, suiting for the collocation method. Their method also requires the transformation of a computational domain of $[-1,1]$ with explicit weighting coefficients within the framework of a collocation method and related to the differential quadrature method (DQM) of Chang [9].

The Chebyshev collocation method's success is continually growing and can be found in the study of Wattanasakulpong and Mao [10] and Wattanasakulpong and Chaikittiratana [11]. In these references, the applications are based on Timoshenko's beam theory.