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Image Segmentation via Fischer-Burmeister Total Variation and Thresholding

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Abstract. Image segmentation is a significant problem in image processing. In this paper, we propose a new two-stage scheme for segmentation based on the Fischer-Burmeister total variation (FBTV). The first stage of our method is to calculate a smooth solution from the FBTV Mumford-Shah model. Furthermore, we design a new difference of convex algorithm (DCA) with the semi-proximal alternating direction method of multipliers (sPADMM) iteration. In the second stage, we make use of the smooth solution and the K-means method to obtain the segmentation result. To simulate images more accurately, a useful operator is introduced, which enables the proposed model to segment not only the noisy or blurry images but the images with missing pixels well. Experiments demonstrate the proposed method produces more preferable results comparing with some state-of-the-art methods, especially on the images with missing pixels.

AMS subject classifications: 65K10, 68U10, 94A08

Key words: Image segmentation, Fischer-Burmeister total variation, difference of convex algorithm, sPADMM, K-means method.

1 Introduction

Image segmentation is to divide an image into distinct objects in reality. In recent years, many great approaches have been proposed to handle this problem [1–7]. In [8], an en-

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ergy minimization approach was proposed by Mumford and Shah named Mumford-Shah model, and their idea is to find a suitable piecewise continuous or constant solution u for a given image f. The Mumford-Shah model has been widely used in various aspects [9–13]. Moreover, its variants have been extensively used in image processing owing to their flexibility and adaptation in numerical implementation. Denote $\Omega\subset\mathbb{R}^2$ as a bounded and continuous open set, $f:\Omega\to\mathbb{R}$ as a given grayscale image. In general, we set $f\in[0,1]$, therefore $f\in L^\infty(\Omega)$. Especially, the energy minimization problem for image segmentation is as follows

$$E(u,\Gamma)=\mathcal{H}^1(\Gamma)+\frac{\mu}{2}\int_{\Omega\backslash\Gamma}|\nabla u|^2dx+\frac{\eta}{2}\int_\Omega(f-u)^2dx, \tag{1.1}$$

where μ, η are two positive constants, $\mathcal{H}^1(\cdot)$ denotes the one-dimensional Hausdorff measure in $\mathbb{R}^2$, Γ is a compact curve in Ω, ∇ represents gradient operator, and $u:\Omega\to\mathbb{R}$ is an underlying piecewise smooth approximation of f. The latent image u is continuous or even derivable in $\Omega\backslash\Gamma$, whereas it is discontinuous across Γ. As aforementioned, Eq. (1.1) is nonsmooth and nonconvex, it is difficult to be solved efficiently. To overcome this shortcoming, some researchers proposed to utilize the discrete functionalities to approximate Eq. (1.1) [8,14,17] and achieve good segmentation results.

Furthermore, a simplified approach [15,16] of Eq. (1.1) has been proposed to improve the Mumford-Shah model. While ∇u is set to be zero identically on the domain $\Omega\backslash\Gamma$, it becomes the Mumford-Shah model with the piecewise constant. In [18], Chan and Vese proposed an active contours strategy without edges, which fixes the solution to be two piecewise constants (Chan-Vese model). However, Chan-Vese model cannot segment the images with intensity inhomogeneity or elongated structures well. Many models related to the Chan-Vese model were presented to solve this problem, we refer to see [19,20].

Recently, Cai, Chan and Zeng [21] proposed an image segmentation method based on the convex variant of the Mumford-Shah model and the thresholding strategy, which can be divided into two stages (TSMS). In the first stage, the authors found a smooth image u from the given image f. The smooth u can be obtained by solving the following total variation (TV) based model

$$\min_{u\in W^{1,2}(\Omega)}\int_\Omega|\nabla u|dx+\frac{\eta}{2}\int_\Omega(f-Au)^2dx+\frac{\mu}{2}\int_\Omega|\nabla u|^2dx, \tag{1.2}$$

where $W^{1,2}(\Omega)$ is the Sobolev space [22]. $u\in W^{1,2}(\Omega)$ is the clean image and A is a blurring or identity operator. In the second stage, a K-means method is applied to threshold the smoothed image obtained in the first stage. The two-stage strategy has been widely used for image segmentation. For example, Chan, Yang and Zeng [23] proposed a two-stage approach to segment the blurry images with Poisson or multiplicative Gamma noise. Duan et al. [24] extended the two-stage segmentation method by applying the Euler's Elastica regularization. Zhi, Sun and Pang [25] presented a two-stage image segmentation scheme based on the Inexact Alternating Direction Method. Ma, Peng and