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Numerical Solution for Neutral Delay Differential Equation of Constant or Proportional Type using Hybrid Block Method

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Abstract. Neutral Delay Differential Equation (NDDE) is a differential problem that has regularly existed in numerous occurrences and has represented a significant role in dealing with real-life phenomena, especially on their application in biological and physiological processes. A fifth-order two-point hybrid implicit multistep block method (2PIH5) has been formulated in this research for the numerical solution of Neutral Delay Differential Equation (NDDE). A Taylor series interpolation polynomial has been implemented in the formulation of the proposed 2PIH5. The order, consistency, and zero-stability for 2PIH5 have been illustrated. The analyses of convergence and stability test have been performed and discussed. The initial value problems for the first-order NDDE with constant or proportional delay have been solved using the proposed block method. Some numerical results for the proposed method have been presented to prove the adaptability and applicability of the proposed method in solving NDDE. The proposed method is proved to be comparable with the other existing methods. It is assumed to be reliable and efficient for solving the first-order NDDE with constant or proportional delay.

AMS subject classifications: 65M10, 78A48

Key words: Constant delay, hybrid method, multistep block method, neutral delay differential equation, proportional delay.

1 Introduction

Neutral Delay Differential Equation (NDDE) represents an essential model in real-life handling phenomena, specifically on its implementation in science engineering. NDDE

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has become a well-known equation to be solved among researchers. As quoted in [21], the delay terms can be displayed as a transport delay. For a comparative example, a neutral type DDE generally appears in the modelling of the coupled oscillatory system, where the oscillators are connected and allow energy motion to be transferred between them. Time delay is considered to illustrate the model realistically, given as follows:

$$y'(x)=f(x,y(x),y(x-\tau_i),y'(x-\tau_i)), \qquad x\geq x_0, \tag{1.1a}$$
$$y(x)=\phi(x), \qquad x\leq x_0, \tag{1.1b}$$
$$y'(x)=\phi'(x), \qquad x\leq x_0, \tag{1.1c}$$

where τ_i is known as the delay, while $y(x-\tau_i)$ and $y'(x-\tau_i)$, for $0\leq i\leq n$, are the expressions of delay solutions and its derivative. Eq. (1.1) is known as a constant type of NDDE since only a constant value of delay is considered. Other than constant NDDE, a proportional delay type also plays an essential role in industrial development. The general form of the proportional equation being modelled in NDDE is shown as follows:

$$y'(x)=f(x,y(x),y(qx),y'(qx)), \qquad x\geq x_0, \tag{1.2a}$$
$$y(x)=\phi(x), \qquad x\leq x_0, \tag{1.2b}$$

where $0<q<1$ and $y(x)=\phi(x)$ is the given initial value. From [5], the function $\phi(x)$ is defined in $[\rho,x_0]$, where

$$\rho=\min_{1\leq i\leq n}\Big\{\min_{x\geq x_0}(x-\tau_i)\Big\},$$

since for some $x\geq x_0$, $x-\tau_i<x_0$. Pantograph equation is named after the pantograph on trains. Usually, an electric train has an apparatus mounted on its roof as a current collector from an overhead line. The delay application is implemented on the time taken for the current to travel from an overhead line to the electronic train or bus. In the advance communication technologies, these transmission systems are frequently transpired, where a command device (a sensor) transmits the feedback to an actuator by converting the signal's energy into mechanical motion.

Several authors have produced suitable methods for the numerical solutions of NDDE. A series of methods for NDDE have been proposed by [14–18] and [19] for both one-step and multistep methods in predictor-corrector Adams method. Some of the authors have proposed semi-analytical methods, such as variational iteration method (VIM), homotopy perturbation method (HPM), reproducing kernel Hilbert space method (RKHSM) and homotopy analysis method (HAM) by [6, 7, 23] and [26], respectively. Those four methods have been compared with two-stage order-one Runge-Kutta method [29] and one-leg θ-method [31]. The results obtained have proved the applicability and suitability of analytical methods in solving NDDE. Then, VIM from [7] has been modified by [8] in controlling the convergence region of approximate solutions for proportional delay type.

Then, [30] have applied the differential transformation method to develop exact and approximate solutions of nonlinear differential and integro-differential equations with