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Stability and Convergence of the Canonical Euler Splitting Method for Nonlinear Composite Stiff Functional Differential-Algebraic Equations

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Abstract. A novel canonical Euler splitting method is proposed for nonlinear composite stiff functional differential-algebraic equations, the stability and convergence of the method is evidenced, theoretical results are further confirmed by some numerical experiments. Especially, the numerical method and its theories can be applied to special cases, such as delay differential-algebraic equations and integral differential-algebraic equations.

AMS subject classifications: 65L03, 65L04, 65L80

Key words: Canonical Euler splitting method, nonlinear composite stiff functional differential-algebraic equations, stability, convergence.

1 Introduction

Functional differential-algebraic equations (FDAEs) have been widely used in science problems in mechanics, control science, biology and other fields [1, 2]. The reference [3] has indicated that differential-algebraic equations (DAEs) are neither differential equations nor algebraic equations, they include the process of differentiation and the limitations of algebraic conditions, which change the behavior of the solution and lead to some difficulties of numerically solving FDAEs.

In recent years, there has been extensively studied on the numerical stability and convergence of delay differential-algebraic equations [4–15]. Further, we can refer to [16–19] for details on the numerical stability and convergence of integral differential-algebraic equations. However, most of the above studies are focused on theoretical and numerical analysis of linear or non-stiff problems, we can refer to [20–22] for the stability of the more

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general nonlinear stiff FDAEs and theirs numerical methods. For these stiff problems, since the right-side functions of the equations exhibit different stiffness at different stages of development, when we solve the stiff problems on the slow-varying interval, although the fast-varying interval has been attenuated to insignificance, but the fast-changing interference will still affect the stability and accuracy of the numerical solution. If it is solved numerically in the entire interval, it will increase the amount of calculation and fail to achieve high accuracy. As a consequence, some scholars have proposed some splitting methods, such as operator splitting method, symmetric weighted sequential splitting method, high-order splitting method, and Strang-Marchuk splitting method, but currently these splitting methods are mainly used to solve differential equations without algebraic constraints, such as the splitting methods for stiff differential equations [23–25], Schrödinger equations [26–33], nonlinear convection-diffusion-reaction equations [34,35], nonlinear delay differential equations and integral differential equations [36–40]. Nevertheless, the splitting methods and their theories mentioned in the above references are aimed at the nonlinear or stiff problems with some special structures, and still cannot be applied to the general nonlinear composite stiff functional differential equations [41].

The canonical Euler splitting method (CES) is proposed for solving nonlinear composite stiff evolution equations [41]. On this basis, in order to effectively overcome the difficulties caused by algebraic constraints, we further propose a new CES method to solve the nonlinear composite stiff FDAEs, and prove the stability and convergence of the CES method, and the numerical experiments verify the theoretical results of the method.

2 Canonical Euler splitting method for solving nonlinear composite stiff FDAEs

Consider the nonlinear composite stiff FDAEs

$$\begin{cases} y'(t)=f(t,y(t),y(\cdot),z(t),z(\cdot)), & t\in(0,T],\\ z(t)=g(y(t),y(\cdot),z(\cdot)), & t\in(0,T],\\ y(t)=\varphi_1(t), \quad z(t)=\varphi_2(t), & t\in[-\tau,0], \end{cases} \tag{2.1}$$

where $T>0$, $\tau\in[0,+\infty]$ are constants, and initial functions φ_1, φ_2 are given, $\mathbf{R}^{m_i}$ represents the m_i dimensional Euclidean space, $i=1,2$, the inner product is denoted as $\langle\cdot,\cdot\rangle$, and the corresponding norm is denoted as $\|\cdot\|$, the mappings

$$\begin{aligned} f:&\ [0,T]\times\mathbf{R}^{m_1}\times\mathbf{C}_{m_1}[-\tau,T]\times\mathbf{R}^{m_2}\times\mathbf{C}_{m_2}[-\tau,T] \ \rightarrow\ \mathbf{R}^{m_1},\\ g:&\ \mathbf{R}^{m_1}\times\mathbf{C}_{m_1}[-\tau,T]\times\mathbf{C}_{m_2}[-\tau,T] \ \rightarrow\ \mathbf{R}^{m_2}, \end{aligned}$$

are given, and the mapping g satisfies the consistency condition at the point $t=0$: $z(0)=g(y(0),\varphi_1(0),\varphi_2(0))$, f can be divided into two sub-mappings

$$f(t,u,\psi(\cdot),v,\chi(\cdot))=f_1(t,u,\psi(\cdot),v,\chi(\cdot))+f_2(t,u,\psi(\cdot),v,\chi(\cdot)),$$