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Numerical Analysis of Two-Grid Block-Centered Finite Difference Method for Two-Phase Flow in Porous Medium

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Abstract. In this paper, a two-grid block-centered finite difference method for the incompressible miscible displacement in porous medium is introduced and analyzed, which is to solve a nonlinear equation on coarse mesh space of size H and a linear equation on fine grid of size h. We establish the full discrete two-grid block-centered finite difference scheme on a uniform grid. The error estimates for the pressure, Darcy velocity, concentration variables are derived, which show that the discrete L_2 error is $\mathcal{O}(\Delta t+h^2+H^4)$. Finally, two numerical examples are provided to demonstrate the effectiveness and accuracy of our algorithm.

AMS subject classifications: 65M06, 65M12, 65M15, 65M55

Key words: Porous media, two phase flow, block-centered finite difference, two-grid, numerical analysis.

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1 Introduction

In this paper, we consider the incompressible miscible displacement in porous media [1–3]

$$\begin{cases} \nabla\cdot u=q(x,t), & x\in\Omega, \quad t\in J,\\ u=-\dfrac{\kappa(x)}{\mu(c)}\nabla p, & x\in\Omega, \quad t\in J,\\ \varphi(x)\dfrac{\partial c}{\partial t}+\nabla\cdot(uc)-\nabla\cdot(D\nabla c)=\tilde{c}q, & x\in\Omega, \quad t\in J. \end{cases}$$

We assume that Ω is a rectangular domain in R^2, $t\in J=(0,T]$, and T denotes the final time. The concentration is denoted by $c(x,t)$, $p(x,t)$ is the fluid pressure, and $u=(u_1,u_2)^T$ is Darcy velocity of the fluid, $\kappa(x)$ and $\varphi(x)$ represent the permeability and porosity of

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the porous medium, respectively. $\mu(c)$ is the concentration dependent viscosity. The function $\tilde{c}$ is the concentration of the same component as measured by c in the injected fluid, which must be specified whenever injection is taking place, and it will be assumed that $\tilde{c}=c$, when the fluid is being produced, q is the external flow rate at wells.

For the sake of simplicity, let $a(c)=\kappa(x)/\mu(c)$, $D=\varphi d_m I=\lambda I$, and $w=(w^x,w^y)=uc-D\nabla c=uc-\lambda\nabla c$, where d_m is the molecular diffusivity, I is the second order identity matrix. Then the question may be equivalently written in the form:

$$\begin{cases} \nabla\cdot u=q(x,t), & x\in\Omega, \quad t\in J, & (1.1a)\\ u=-a(c)\nabla p, & x\in\Omega, \quad t\in J, & (1.1b)\\ \varphi(x)\dfrac{\partial c}{\partial t}+\nabla\cdot w=f(\tilde{c}), & x\in\Omega, \quad t\in J, & (1.1c)\\ w=uc-\lambda\nabla c, & x\in\Omega, \quad t\in J, & (1.1d)\end{cases}$$

where $f(\tilde{c})=\tilde{c}q$.

We consider the following boundary condition and initial condition for the problem:

$$u\cdot\mathbf{n}=0, \qquad x\in\partial\Omega, \quad t\in J, \tag{1.2a}$$

$$(\lambda\nabla c)\cdot\mathbf{n}=0, \qquad x\in\partial\Omega, \quad t\in J, \tag{1.2b}$$

$$c|_{t=0}=c_0, \qquad x\in\Omega, \tag{1.2c}$$

where $\mathbf{n}$ is the unit outward normal vextor to $\partial\Omega$, the compatibility condition and the uniqueness condition are as follows

$$\int_\Omega p(x)\mathrm{d}x=0. \tag{1.3}$$

For problem (1.1a)-(1.3), we consider the following smoothness hypotheses (H):

(1) The funtions $a(c)$,$b(c)$,λ are bounded, And, there exist positive constants a_0, a_1, b_0, b_1, λ_0, λ_1, such that

$$0<a_0\le a\le a_1, \quad 0<b_0\le b\le b_1, \quad 0<\lambda_0\le\lambda\le\lambda_1.$$

(2) The second derivative of f, q are continuously bounded in $\Omega\times J$, f, a is Lipschitz-continuous corresponding to variable c. The function φ is continuous, there exist a positive constant φ_0, such that $\varphi\ge\varphi_0>0$.

(3) $p\in L_\infty(J;W^4_\infty(\Omega))$, $u\in C^1(J;W^1_\infty(\Omega))^2$, $c\in W^2_\infty(J;W^4_\infty(\Omega))$.

People have been interested in efficient oil exploitation and improving the utilization of groundwater resource for a long time. Two-phase flow and transportation of fluids in porous media play a vital role in both theoretic and applicative aspects in groundwater contamination or petroleum engineering. The incompressible miscible displacement