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Numerical Analysis of Crack-Tip Fields using a Meshless Method in Linear Elastic Materials

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Abstract. In this paper, the Discrete Least Squares Meshless (DLSM) method is developed to determine crack-tip fields. In DLSM, the problem domain and its boundary are discretized by unrelated field nodes used to introduce the shape functions by the moving least-squares (MLS) interpolant. This method aims to minimize the sum of squared residuals of the governing differential equations at any nodal point. Since high-continuity shape functions are used, some necessary treatments, including the visibility criterion, diffraction, and transparency approaches, are employed in the DLSM to introduce strong discontinuities such as cracks. The stress extrapolation and J-integral methods are used to calculate stress intensity factors. Three classic numerical examples using three approaches to defining discontinuities in the irregular distribution of nodal points are considered to investigate the effectiveness of the DLSM method. The numerical tests indicated that the proposed method effectively employed the approaches to defining discontinuities to deal with discontinuous boundaries. It was also demonstrated that the diffraction approach obtained higher accuracy than the other techniques.

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Key words: Numerical analysis, partial differential equations, fracture mechanics, meshless method, discrete least-squares, modification of weight function, approaches to defining discontinuities, stress intensity factors.

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1 Introduction

Structural defects such as micro-cracks always have an adverse effect on their service life. Therefore, it is essential to investigate crack-tip fields to determine the safety factors and predict the service life of mechanical structures. Recent years have seen substantial growth in the use of fracture mechanics in structural analysis and design. Crack problems can be analyzed as elliptic partial differential equations (PDEs) whereby crack-tip stress singularities can be explained, which is determined by the stress intensity factor (SIF).

Analytical solutions can be used to solve crack problems with regular, non-complex boundaries in infinite planes. However, numerical methods must be applied to solve various fracture mechanics problems with complex geometric configurations and loading conditions. The finite element method (FEM) is employed as usual to solve fracture mechanics problems. The domain meshing-based methods, e.g., FEM, have some shortcomings in calculating fracture mechanic parameters, including failure to accurately identify near-crack-tip singularities [1] and limited ability to model the crack growth. The FEM requires mesh modification and updating to simulate the crack growth, a time-consuming and costly process. The boundary element method (BEM) [2] was applied to solve crack problems, as it is a time-efficient, sufficiently accurate method that only requires boundary discretization. The extended finite element method (XFEM) [3] is an improved version of the FEM that is appropriately designed for fracture mechanics problems [4]. The XFEM has certain advantages over the conventional FEM; for example, it can model cracks with arbitrary geometric shapes independently of the FEM meshes, and it requires minimal re-meshing in solving crack growth problems. Although some methods, e.g., node release, are introduced to overcome the existing problems [5,6], some challenges still exist.

The problems detailed above and other shortcomings such as strain/stress discontinuity on element surfaces and the need for some additional operations for smoothing the results have motivated researchers to try other numerical methods [1]. Thus, several meshless methods are introduced to overcome these problems. Meshless methods are developed under two branches of formulations: weak form and strong form. The problem domain is discretized using nodal points in both methods. However, weak-form meshless methods require background meshes to obtain Gauss points for integration despite the higher relative accuracy of the results. In some cases, these meshless methods incur a higher computational cost than mesh-based methods, where integration problems at complex boundaries still persist. Strong-form meshless methods, on the other hand, directly solve PDEs and reduce the computational cost. However, they have certain disadvantages, such as instability, low accuracy of the results, difficulty in applying boundary conditions, and asymmetric coefficient matrix.

The application of smooth interpolants in meshless methods has led to the desired results, rendering them advantageous over the FEM. Besides, they have outperformed the FEM in solving problems with moving boundaries, large deformation, and crack propagation. Despite these advantages, meshless methods also have some disadvantages, in-