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A Direct-Forcing Immersed Boundary Projection Method for Thermal Fluid-Solid Interaction Problems

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Abstract. In this paper, we develop a direct-forcing immersed boundary projection method for simulating the dynamics in thermal fluid-solid interaction problems. The underlying idea of the method is that we treat the solid as made of fluid and introduce two virtual forcing terms. First, a virtual fluid force distributed only on the solid region is appended to the momentum equation to make the region behave like a real solid body and satisfy the prescribed velocity. Second, a virtual heat source located inside the solid region near the boundary is added to the energy transport equation to impose the thermal boundary condition on the solid boundary. We take the implicit second-order backward differentiation to discretize the time variable and employ the Choi-Moin projection scheme to split the coupled system. As for spatial discretization, second-order centered differences over a staggered Cartesian grid are used on the entire domain. The advantages of this method are its conceptual simplicity and ease of implementation. Numerical experiments are performed to demonstrate the high performance of the proposed method. Convergence tests show that the spatial convergence rates of all unknowns seem to be super-linear in the 1-norm and 2-norm while less than linear in the maximum norm.

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Key words: Fluid-solid interaction, heat transfer, direct-forcing method, immersed boundary method, projection scheme.

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1 Introduction

In this paper, we will develop a direct-forcing immersed boundary approach combined with the Choi-Moin projection scheme for numerical simulations of heat transfer in the thermal fluid-solid interaction (TFSI) problems. We particularly emphasize the accurate imposition of the Neumann thermal boundary condition on the immersed solid boundary. It is well known that the understanding of the interaction dynamics between fluid and structure is of great importance in many applications of science and engineering. A commonly used numerical approach for simulating the flow dynamics with complex boundaries is based on the body-fitted discretization. The incompressible Navier-Stokes equations are spatially discretized over an unstructured grid that conforms to the immersed structure boundaries, and thus the boundary conditions can be imposed directly. However, due to the geometrical complexity encountered in the problems, it is still challenging and computationally expensive to simulate the dynamics of fluid-structure interaction problems using the conventional body-fitted approach.

To efficiently address the complex fluid-solid interaction (FSI) problems, one often appeals to the Cartesian grid-based non-boundary conforming methods. Such kind of techniques based on Cartesian grids provides many advantages over the body-fitted methodology, such as simplicity in grid generation, savings in computation time and memory usage, and straightforward parallelization. Consequently, they have been extensively studied in the analysis of FSI problems in science and engineering applications. In the past decades, the so-called immersed boundary (IB) method, which was first developed by Peskin [30, 31] in the 1970s, is one of the powerful Cartesian grid-based methods that is frequently used for simulating the dynamics of FSI problems, even with moving boundaries. In the IB method, the immersed structure exerts a boundary force on the fluid, and the interaction between structure and fluid can be represented by a contribution to the forcing term in the fluid equations by means of the Dirac delta function. Instead of generating a boundary-fitted grid to the immersed boundary at each time step, the spatial discretization of the IB method is implemented over Cartesian grids for the entire domain, and the immersed boundary is discretized by a set of Lagrangian marker points that are not constrained to lie on the grids. For more details we refer the reader to e.g., [18,24,31] and many references cited therein. This paper aims to study a popular variant of the IB method, called the direct-forcing IB method, see e.g., [3, 4, 8, 9, 12, 14–17, 20–23, 25, 27–29, 33–36, 38, 40]. We note that most of the direct-forcing IB methods reported in the literature have been directed toward analyzing fluid flow problems. However, in the present work, we are mainly concerned with heat transfer phenomena in TFSI problems.

In this paper, we will develop an efficient direct-forcing IB projection method for simulating heat transfer in the TFSI problems. This direct-forcing approach was first considered by Kajishima et al. [16, 17] in the 2000s for FSI problems and has been further studied in many applications, see, e.g., [3,14,27], and references cited therein. The underlying idea of this direct-forcing approach for addressing the TFSI problems is that we first