Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **15**, No. 1, pp. 69-93

DOI: 10.4208/aamm.OA-2022-0048
February 2023

Multi-Modes Multiscale Approach of Heat Transfer Problems in Heterogeneous Solids with Uncertain Thermal Conductivity

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Received 23 February 2022; Accepted (in revised version) 19 April 2022

Abstract. Stochastic temperature distribution should be carefully inspected in the thermal-failure design of heterogeneous solids with unexpected random energy excitations. Stochastic multiscale modeling for these problems involve multiscale and high-dimensional uncertain thermal parameters, which remains limitation of prohibitive computation. In this paper, we propose a multi-modes based constrained energy minimization generalized multiscale finite element method (MCEM-GMsFEM), which can transform the original stochastic multiscale model into a series of recursive multiscale models sharing the same deterministic material parameters by multiscale analysis. Thus, MCEM-GMsFEM reveals an inherent low-dimensional representation in random space, and is designed to effectively reduce the complexity of repeated computation of discretized multiscale systems. In addition, the convergence analysis is established, and the optimal error estimates are derived. Finally, several typical random fluctuations on multiscale thermal conductivity are considered to validate the theoretical results in the numerical examples. The numerical results indicate that the multi-modes multiscale approach is a robust integrated method with the excellent performance.

AMS subject classifications: 65N12, 65N15, 80M10, 80M22
Key words: Stochastic multiscale heat transfer problems, uncertainty quantification, MCEM-GMsFEM, multimodes expansion.

1 Introduction

Heterogeneous solids are widely used in engineering practice, and are often exposed to strong temperature changes, such as thermal protection systems for space aircraft [1]

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and thermal coating for microelectronic systems [2], etc. Under severe thermal conditions, accurate and efficient prediction of the damage or fracture process of heterogeneous solids requires a comprehensive understanding of the uncertainty propagation of temperature fields under a multiscale framework [3, 4], where stochasticity comes from morphological randomness or material uncertainty [5], such as mismatches between the micro components due to their dispersion and orientation, stochastic cracks or defects, measurement errors or incomplete cognition of thermal expansion and conduction parameters [6]. Therefore, developing a new reliable and effective uncertainty quantification method for multiscale heat transfer problem [7, 8] is of paramount importance and practical significance, and this method can both capture all stochastic responds with high efficient simulations and alleviate the computational cost.

Accounting for the uncertainty is a type of classic issue in the numerical computation of random medium [9]. Due to the high-dimensional nature of the random space, many theories have been developed to deal with the problems. Monte Carlo method and its variants [10] are usually used to solve the governing equation of stochastic problems, in which the uncertainty is sampled by use of the probability distribution function or the random field, and then the corresponding stochastic problem is approximated by a series of deterministic problems, statistical information of their exact solutions can be obtained subsequently. Another typical methods are to approximate the physical quantities of interest in the random space and physical space, respectively. These include the stochastic Galerkin method [11, 12] and stochastic collocation method [13, 14], etc. The uncertainty is generally represented in the random space through Wiener chaos expansion, generalized polynomial chaos expansion or collocation points. Here, the stochastic collocation method combining the advantages of stochastic finite element method and Monte Carlo sampling, has received wide attention, where the collocation points can be chosen with full tensor product method [15] and Smolyak sparse grid method [13] etc. In [16–20], the multi-modes Monte Carlo method is successfully applied to solve various important stochastic problems. Through these methods, the computation saving is obtained by the dimensinality reduction. It should be pointed out that the multiscale modeling and computational method of stochastic heat conduction problem in the high-dimensional uncertainty space is still extremely challenging due to their complex correlative nature.

Except for uncertainties of thermal conductivity, it is also necessary to consider its multiscale features in heterogeneous solids. This leads to tremendous cost for solving the stochastic multiscale heat transfer problem by use of traditional numerical methods. Therefore, numerous researchers have begun to pay attention to the design of the multiscale models and computation methods, which can efficiently reduce the complex fine-scale problems to coarse-scale problems, including homogenization method [21–24], variational multiscale method (VMM) [25–28], upscaling method [29–31], heterogeneous multiscale methods (HMM) [32], and multiscale finite element method (MsFEM) [33–39]. Moreover, when the uncertainties are integrated into the multiscale model, some attempts have been made to deal with the coupling of the multiscale and uncertainty characteristics of stochastic multiscale problem [26, 40–44]. [45] proposed a stochastic mul-