Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **15**, No. 1, pp. 94-117

DOI: 10.4208/aamm.OA-2021-0314
February 2023

A Two-Stage Color Image Segmentation Method Based on Saturation-Value Total Variation

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Received 6 October 2021; Accepted (in revised version) 22 May 2022

Abstract. Color image segmentation is crucial in image processing and computer vision. Most traditional segmentation methods simply regard an RGB color image as the direct combination of the three monochrome images and ignore the inherent color structures within channels, which contain some key feature information of the image. To better describe the relationship of color channels, we introduce a quaternion-based regularization that can reflect the image characteristics more intuitively. Our model combines the idea of the Mumford-Shah model-based two-stage segmentation method and the Saturation-Value Total Variation regularization for color image segmentation. The new strategy first extracts features from the color image and then subdivides the image in a new color feature space which achieves better performance than methods in RGB color space. Moreover, to accelerate the optimization process, we use a new primal-dual algorithm to solve our novel model. Numerical results demonstrate clearly that the performance of our proposed method is excellent.

AMS subject classifications: 65M10, 78A48

Key words: Color space, pure quaternion, image segmentation, total variation, primal-dual algorithm.

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1 Introduction

Color image segmentation is an important and challenging task in image processing and computer vision. The goal of image segmentation is to divide an image into different parts with similar characteristics. This process has vast applications in many fields, such as biomedical imaging [44], object detection [23,50], and astronomical imaging [41], etc. In the literature, many excellent methods have been proposed for image segmentation [37, 39, 40, 49, 54]. One of the most effective and significant segmentation models is

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the Mumford-Shah model (MS) [40]. Let $\Omega\subset\mathbb{R}^2$ be a bounded open connected set, and Γ be a compact curve in Ω, the MS model can be formulated as

$$E_{\rm MS}(u,\Gamma;\Omega)=\frac{\lambda}{2}\int_\Omega (f-u)^2dx+\frac{\mu}{2}\int_{\Omega\backslash\Gamma}|\nabla u|^2dx+\text{Length}(\Gamma), \tag{1.1}$$

where λ, μ are positive parameters, $f:\Omega\rightarrow\mathbb{R}$ is the degraded image and $u:\Omega\rightarrow\mathbb{R}$ is the ideal image. Here the length of Γ can also be written as $\mathcal{H}^1(\Gamma)$, i.e., the 1-dimensional Hausdorff measure in $\mathbb{R}^2$. It is challenging to approximate the solution of Model (1.1) since it is a nonconvex minimization problem.

Over the years, plenty of methods [1,2,11,12,25,38] have been proposed to minimize or approximate the Mumford–Shah energy (1.1). Meanwhile, in order to overcome the computational complexity, numerous models and approaches related to the Mumford-Shah Model are proposed, such as the famous Chan-Vese Model [16], convex relaxation approaches [6, 15, 42] and multiphase segmentation algorithms [3, 7, 22, 33, 35, 54], etc. In [10], a two-stage image segmentation strategy for gray-scale images was proposed and it performed well on the segmentation task. For this method, the first stage is to find a smooth image u by minimizing the following energy:

$$\inf_u\left\{\frac{\lambda}{2}\int_\Omega (f-\mathcal{A}u)^2dx+\frac{\mu}{2}\int_\Omega|\nabla u|^2dx+\int_\Omega|\nabla u|dx\right\}, \tag{1.2}$$

where $\mathcal{A}$ can be the identity operator for noisy observed image f or a blurring operator for the blurred case. Then the second stage is to segment by thresholding u properly. Base on the two-stage idea, many novel image segmentation methods [14, 20, 24, 34, 51] have been proposed. They show the close correlation between image restoration and segmentation to some extent and all these two-stage methods have achieved good results in image segmentation.

Conceiving an effective color image segmentation method is not a simple extension of approaches for gray images, because both the brightness and chrominance information need to be considered. Most methods are proposed in the usual RGB color space [8, 30, 43, 46], but it has been observed that the RGB color space is not suitable for segmentation since it often fails to reflect features of real-world color images due to the lack of correlation between the three channels of RGB space. In [45], RGB image is converted to the HSI (Hue, Saturation and Intensity) color space for segmentation. In [4], a segmentation method for grayscale images is extended to color images in RGB, HSV (hue, saturation and value) and CB (chroma-brightness) color spaces. It shows that color image in the HSV color spaces is similar to human color perception, and seems to be grouped more easily than in RGB color space (see Fig. 2 for illustration). In this paper, we will further showcase through experiments that image segmentation using features in the HSV space is valid and effective.

An effective way to represent color images is to use quaternions. Many previous works [18, 19, 52, 53] employ quaternions to express the color images. Among these