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An Analytical Approach for Nonlinear Buckling Analysis of Torsionally Loaded Sandwich Carbon Nanotube Reinforced Cylindrical Shells with Auxetic Core

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Abstract. The main aim of this paper is to present an analytical approach on the postbuckling for torsionally loaded sandwich carbon nanotube (CNT) reinforced cylindrical shells with the auxetic core. The considered shells consist of three layers, external and internal CNT reinforced layers, and the auxetic lattice core made by isotropic material. The homogenization model for honeycomb auxetic lattice core is utilized, and the equilibrium equations are formulated based on the nonlinear Donnell's thin shell theory with von Kármán geometrical nonlinearities. The three terms of deflection are considered, the Airy's stress function and Galerkin's method are utilized, the explicit expression of critical buckling of torsionally loaded shells and load-deflection expression of postbuckling states are achieved. The effects of two carbon nanotube reinforced layers, the auxetic core layer, the volume fraction of carbon nanotube on the torsional buckling behavior are examined and remarked.

AMS subject classifications: 74K25, 74G60

Key words: Nonlinear postbuckling, carbon nanotube reinforced composite, cylindrical shell, torsion, auxetic core.

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1 Introduction

An advanced composite material mentioned much in recent years is functionally graded material (FGM). Thermo-mechanical behavior of FGM cylindrical shells is a vibrant topic that attracts a lot of interest from authors around the world. Many different aspects of these problems, especially the torsional buckling problem, were analyzed comprehensively, such as statically nonlinear and linear torsional buckling problems without stiffeners [1–3] and with stiffeners [4, 5], torsional dynamic buckling, and vibration without elastic foundation [6] and with elastic foundation [7, 8]. In addition, spiral stiffened FGM cylindrical shells were mentioned [9–11] and its statically nonlinear buckling behavior was studied by improving the smeared stiffener technique. The hierarchical stiffener design for cylindrical shells was optimized using a hybrid optimization method [12]. The FGM and porous FGM cylindrical long shells were also performed with the conveying fluid [13] and in micro scale [14].

Carbon nanotube (CNT) is the special form of carbon allotrope in the form of a grid tube that possesses many outstanding thermo-electro-mechanical properties, which leads to their use in a large number of applications. With the very large elastic moduli, lightweight, small thermal expansion coefficient, excellent conductivity, excellent optical transparency et al., CNT is the perfect material to reinforce the basic material matrix in the composite manufacturing procedure [15–34].

The excellent thermo-mechanical properties of carbon nanotube-reinforced composite (CNTRC) depend on the distribution law of CNT in the basic material and the linkage capacity between the CNT and matrix. In recent years, the thermo-mechanical buckling strength of the CNT-reinforced beams, plates and cylindrical shells was focused on by many authors in the world. The behavior of wave propagation, vibration and dynamic stability of CNT-reinforced beams and plates was investigated with the porosities [15], with piezoelectric layers [16], and with different CNT distributions [17]. Static postbuckling strength of axially-loaded, pressure-loaded, torsionally loaded, and magnetically-loaded CNT-reinforced cylindrical shells were presented [18–21] with higher-order shear strains. Besides, the vibration problems of this structure were also mentioned under simply dynamical loads [22–25] and earthquake dynamic loads [26, 27]. The cylindrical shells with orthogonal CNT-reinforced directions and statically nonlinear buckling strength were investigated [28, 29] and the conical shells, more general case of CNT-reinforced and graphene reinforced cylindrical shells, were also mentioned [30–34]. In addition, CNT is also an excellent reinforcing material for concrete structures [35]. The torsional buckling and postbuckling of classical composite and graphene reinforced composite laminated cylindrical shells were also investigated [36, 37].

The auxetic structure is the lightweight structure with an extreme capacity in absorbing impact impulses, thus, it is especially interesting in many modern technologies. In addition, the auxetic structure is the light core that is designed for the sandwich structure to increase the eccentricities of the two bearing layers. Therefore, it is also of great importance in the case of the problem of structures under static loads. With a small increase in