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A Compact Eulerian Interface–Capturing Algorithm for Compressible Multimaterial Elastic–Plastic Flows with Mie–Grüneisen Equation of State

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Abstract. This paper presents an Eulerian diffuse-interface method using a high-order compact difference scheme for simulating elastic-plastic flows with the Mie–Grüneisen (MG) equation of state (EoS). For simulations of multimaterial problems, numerical errors were generated in the material discontinuities owing to inconsistent treatment of the convective terms. Based on the normal-stress-based mechanical equilibrium assumption for elastic-plastic solids, we introduce an improved form of the consistent localized artificial diffusivity (LAD) method to ensure an oscillation-free interface for velocity and normal stress. The proposed algorithm uses a hyperelastic model. A mixture type of the model system was formed by combining the conservation equations for the basic conserved variables, an equation of a unified deviatoric tensor describing solid deformation, and an additional set of equations for solving the material quantities in the MG EoS. Several one- and two-dimensional problems with various discontinuities, including the elastic-plastic Richtmyer–Meshkov instability, were considered for testing the proposed method.

AMS subject classifications: 74F10, 76T99, 74S20

Key words: Multimaterial elastic-plastic flow, Eulerian solid-dynamics, high-order accurate schemes, compact finite difference, localized artificial diffusivity.

1 Introduction

Multiple interactions occur between compressible solid and fluid components in various engineering problems including high-velocity impacts, explosive-structure interactions,

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and inertial confinement fusion. One of the main difficulties in the numerical simulation of these problems is the robust and accurate tracking of the interface between different materials. For problems involving large deformations or topological changes in the solids, Lagrangian and arbitrary Lagrangian–Eulerian (ALE) frameworks often fail owing to mesh distortion. Consequently, severe restrictions must be imposed on the time step. In contrast, Eulerian methods solve these problems with a fixed grid and are hence better suited to such simulations.

In solid-fluid interaction problems, the governing equations of solid dynamics can be classified into two formulations: hypoelastic and hyperelastic. The former employs a stress measure (e.g., the Cauchy stress tensor), whereas the latter utilizes a strain measure (e.g., the deformation gradient tensor). In an effort to address hydrodynamic-like deformations, Eulerian solids with a hypoelastic formulation have been widely implemented in numerical simulations [15,16,27]. In addition, simulations based on hyperelastic Eulerian models have garnered much research attention (cf. [18,28,33,34,36,41,42]). Recently, Peshkov et al. [32] compared the hyperelastic GPR model [18] against the well-known hypoelastic Wilkins model [43].

In the Eulerian framework, the simulation of multiphase elastic-plastic flow is typically classified into two types of methods: interface-tracking and interface-capturing methods. Interface-tracking methods track material interfaces using an indicator function (e.g., the volume-of-fluid method [28], and level-set method [7]. The interface-tracking method maintains a sharp interface but becomes geometrically complex and computationally expensive for problems involving large deformations, especially in three dimensions. In contrast, the interface-capturing method smears the interface over a small number of grid points, converting the interface into an artificial mixing zone. The interface-capturing method is more capable of solving problems involving large deformations in solids. The first attempt to use the interface capturing method and hyperelastic model to solve multimaterial solid dynamics was presented by Favrie et al. [13], and it was further developed and applied to elastic-plastic deformations in [2,12,19,29]. In recent years, Cook et al. [10] developed a hyperelastic model using a single deformation tensor to describe the deformation of each component. Such a model significantly reduces the number of DOFs but also leads to unacceptable errors around the interface when the density gradient is very large [6]. To resolve this problem, Barton [6] developed a new interface-capturing model for multimaterial problems using a single deviatoric strain tensor shared by each component of the material.

In the interface-capturing method, spurious oscillation errors occur near the interface if there is no special treatment near the interface in the numerical method [1]. Such spurious oscillation is attributed to the discontinuity of material properties entering the equation of state (EoS). In numerical simulations, such discontinuities violate the equilibrium conditions at the interface and generate pressure, velocity, or temperature oscillations. For simulations of multimaterial fluids, such oscillation errors were first highlighted and fixed by Abgrall [1]. This work of Abgrall was extended by Shyue [37–39] to cover gas-liquid and gas-solid systems using modified van der Walls and Mie–Grüneisen EoS. An