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Critical Transition Reynolds Number for the Incompressible Flat-plate Boundary Layer as Searched by Numerical Simulation

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Abstract. The critical transition Reynolds number is the lowest value at which the turbulent flow can hold in real flows. The determination of the critical transition Reynolds number not only is a scientific problem, but also is important for some engineering problems. However, there is no available theoretical method to search the critical value. For the hypersonic boundary layer with significant importance for engineering problems, there is no available experimental method to search the critical value so far. Consequently, it is imperative to take numerical method to search it. In this paper, direct numerical simulations (DNS) method is employed to determine the critical transition Reynolds number for the incompressible flat-plate boundary layer. Firstly, under the assumption of parallel flow, the temporal mode DNS is performed to determine the critical value as $Re_{xpcr}=43767$, which is quite close to the numerical results of other people. Secondly, under the condition of nonparallel flow, the spatial mode DNS is performed to determine the critical transition Reynolds number as $Re_{xcr}=3\times10^5$, which is well consistent with the experimental results. In principle, the proposed method in this paper can be extended to the supersonic/hypersonic boundary layer, and that problem will be discussed in the subsequent papers.

AMS subject classifications: 76E09, 76F06

Key words: Critical transition Reynolds number, flat-plate boundary layer, DNS, temporal mode, spatial mode.

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1 Introduction

The concept of critical transition Reynolds number came from the famous pipe experiment of Reynlods [1], in which he found two distinct regimes of flow, i.e., laminar and turbulent flow regimes. If Reynolds number is lower than the critical value, turbulent flow can't hold no matter how strong the environmental disturbances are, and the flow must be laminar. The critical transition Reynolds number for the pipe flow was found as 2040 in the experiment [1], and the Reynolds number is based on the diameter of the pipe and the mean velocity of the cross section. For the channel flow, the critical value was slightly higher than 1000 in the experiments [2–5], and the Reynolds number was calculated by using the half channel width and the mean velocity at the center. For the incompressible flat-plate boundary layer, the critical Reynolds number $\mathrm{Re}_x = U_\infty^* x^*/\nu^*$ was measured as 3×10^5 in the experiments (see Fig. 1) [6, 7], where U_∞^* represents the freestream velocity, x^* is the distance from the leading edge, and ν^* is the kinematic viscosity. It is difficult to carry out the relevant experiment to measure the critical transition Reynolds number for the compressible flows, especially for the supersonic/hypersonic flows. Although there are some experimental results on supersonic flow transition at low Reynolds numbers, the critical value is still unknown. The transition of a supersonic jet at Mach number 2.0 was observed at Reynolds number down to 10^5 in the experiment of Ozawa et al. [8], and the Reynolds number is based on the diameter of the nozzle exit and the jet velocity at the nozzle exit. Fig. 2 is a sketch in a textbook on fluid mechanics, showing the relationship between the Reynold number Rex at the transition location in the incompressible boundary layers and the freestream turbulent intensity Tu [9], and the data in the figure are from many experiments. For the low level of freestream turbulent intensity, as Tu increases, the transition Reynolds number decreases significantly. However, if the freestream turbulent intensity is high enough, the transition Reynolds number tends to be a constant, i.e., the critical transition Reynolds number. This implies that the transition can't take place in the flow at a Reynolds number lower than the critical value, no matter how strong the environmental disturbances are, and that the flow must be laminar.

There are two kinds of critical Reynolds numbers for shear flows. One is the critical transition Reynolds number mentioned above, the other one is the critical Reynolds number of instability calculated by the linear stability theory (LST), i.e., the lowest Reynolds number at which the unstable perturbation can exist in a laminar flow. The critical Reynolds numbers of instability for common shear flows have been found by LST, and have been verified by experiments. For example, it is $+\infty$ for the pipe flow [10], implying that there is no unstable perturbation, and it is 5772 for the channel flow [11]. For the incompressible boundary layer, the critical Reynolds number Re_δ is 519 [12] while taking the displacement thickness of the boundary layer as the reference length, which is corresponding to another Reynolds number $\mathrm{Re}_x \approx 90965$ due to the relationship between Re_δ and Re_x in [13]. The two kinds of critical Reynolds numbers are listed in Table 1, which are quite different from each other. This is because the critical Reynolds number