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A Novel Wavelet-Homotopy Galerkin Method for Unsteady Nonlinear Wave Equations

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Abstract. The Coiflet wavelet-homotopy Galerkin method is extended to solve unsteady nonlinear wave equations for the first time. The Korteweg-de Vries (KdV) equation, the Burgers equation and the Korteweg-de Vries-Burgers (KdVB) equation are examined as illustrative examples. Validity and accuracy of the proposed method are assessed in terms of relative variance and the maximum error norm. Our results are found in good agreement with exact solutions and numerical solutions reported in previous studies. Furthermore, it is found that the solution accuracy is closely related to the resolution level and the convergence-control parameter. It is also found that our proposed method is superior to the traditional homotopy analysis method when dealing with unsteady nonlinear problems. It is expected that this approach can be further used to solve complicated unsteady problems in the fields of science and engineering.

AMS subject classifications: 76B15, 65Mxx, 76Mxx

Key words: Coiflet wavelet, homotopy analysis method, wavelet-homotopy method, wave equations, unsteady.

1 Introduction

Various types of unsteady nonlinear phenomena exist in scientific researches, engineering applications and social life, such as vibration and large deflection of beams and plates, large swing of a pendulum, the Butterfly effect, and wave motion. These time-dependent nonlinear phenomena can be described and characterized by unsteady nonlinear mathematical models. Accordingly, studies on computational methods of these unsteady nonlinear equations have theoretical guiding significance for the nonlinear problems in practical engineering systems. As a recently developed analytical computation technique, the homotopy analysis method (HAM) [1–3] has been recognized as a powerful tool to solve nonlinear equations with strong non-linearity. Although the HAM technique has been

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successfully applied to nonlinear problems in many fields, there are still some key difficulties and problems affecting convergence speed and computational efficiency in practical application. The main issue is that this approach is based on exact computation, which leads to more and more complex polynomials and/or function expressions involved in high-order computation. As a result, it is very difficult to give accurate solutions for complicated nonlinear problems, especially unsteady ones. To overcome this deficiency, several attempts have been made, such as optimization of auxiliary convergence parameters [4], adoption of best matching linear operator [5], truncation of higher-order residual terms [6], and combination the HAM idea with other computational methods such as the finite homotopy-difference method [7] and the general boundary element method [8].

On the other hand, the wavelet analysis [9] has developed rapidly in recent years and has been widely used in various scientific and engineering fields such as signal analysis, image processing, quantum mechanics, medical imaging and diagnosis, seismic exploration data processing, numerical calculation and so on. Illustrative applications include analysis of time-varying signals [10], multi-resolution scheme of image and audio signals [11], electrocardiogram analysis [12] and description of multi-scale structure of turbulence [13]. The early attempt of the wavelet technique in numerical computation can be traced back to the early 1990s [14]. In solving differential equations, when wavelet is selected as the basis function, the most important properties are compactness, smoothness, orthogonality and interpolation. At present, Daubechies orthogonal wavelet, which is widely used, achieves the best balance between compactness and smoothness, but it does not have the property of interpolation. Based on the Daubechies wavelet, Wang [15] constructed a class of orthogonal quasi interpolation wavelets, namely generalized orthogonal Coiflet wavelets. The principle of the wavelet-based method is to use the finite space composed of wavelet basis function to approximate the infinite dimensional function space, and then combine the variational method with the approximate method to obtain the numerical approximate solution. It has known that the wavelet series is the basis function of square integrable space defined on the whole real number axis, while the actual problem is defined on a finite interval. This leads to that most boundary conditions are difficult to be satisfied naturally. Three major difficulties are encountered when the wavelet technique is employed in numerical computation, the first is the boundary jump problem, the second is the treatment of nonlinear term, and the third is the solution of irregular region. Several efforts have been done to solve these issues. For example, Wang [15] proposed a boundary continuation approach similar to Taylor series expansion to reduce the instability near the boundary. The second problem is to deal with the nonlinear terms in the equation. Wang [15] introduced the Galerkin method into the wavelet technique to improve on the capability of treatment of nonlinear equations. Liu et al. [16, 17] proposed a wavelet multiresolution interpolation Galerkin method for solving elasticity problems with irregular domains. Owing to the excellent local characteristics and highly accurate expression capability of the wavelet basis, the wavelet technique has rapidly developed into the cutting-edge technology of nonlinear science. Various wavelet-based computational methods have been developed by combination of