Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **15**, No. 5, pp. 1290-1314

DOI: 10.4208/aamm.OA-2021-0016
October 2023

On the Convergence of a Crank-Nicolson Fitted Finite Volume Method for Pricing European Options under Regime-Switching Kou's Jump-Diffusion Models

Xiaoting Gan[1,*], Junfeng Yin[2] and Rui Li[3]

[1] *School of Mathematics and Computer Science, Chuxiong Normal University, Chuxiong, Yunnan 675000, China*
[2] *School of Mathematical Sciences, Tongji University, Shanghai 200092, China*
[3] *College of Mathematics Physics and Information Engineering, Jiaxing University, Jiaxing, Zhejiang 314001, China*

Received 21 January 2021; Accepted (in revised version) 24 October 2021

Abstract. In this paper, we construct and analyze a Crank-Nicolson fitted finite volume scheme for pricing European options under regime-switching Kou's jump-diffusion model which is governed by a system of partial integro-differential equations (PIDEs). We show that this scheme is consistent, stable and monotone as the mesh sizes in space and time approach zero, hence it ensures the convergence to the solution of continuous problem. Finally, numerical experiments are performed to demonstrate the efficiency, accuracy and robustness of the proposed method.

AMS subject classifications: 65M08, 65M12, 65M60, 91G60

Key words: European option pricing, regime-switching Kou's jump-diffusion model, partial integro-differential equation, fitted finite volume method, Crank-Nicolson scheme.

1 Introduction

To reflect the volatility clustering effect observing in the financial markets, since Hamilton [1,2] introduced the regime-switching options models (see, also, Naik [3], Buffington and Elliott [4]), the pricing of this type of options has been attracting much attention from both mathematicians and financial engineers [5–7]. Actually, the regime-switching framework is the market that may switch from time to time among different regimes, which allows one to account for certain periodicity caused by, e.g., short-term political or economic uncertainty. Further, econometric analysis has shown that the incorporation of a regime switching component with the log-normal dynamics of stock price can

*Corresponding author.
Emails: 9xtgan@alumni.tongji.edu.cn (X. Gan), yinjf@tongji.edu.cn (J. Yin), lirui@tongji.edu.cn (R. Li)

better fit the asset price dynamics [8]. Due to the economic intuition and inexpensive computation, regime-switching model has also been widely applied in other fields, e.g., valuation of stock loans [9], natural gas [10], forestry valuation [11], etc. However, compared with the standard Black-Scholes pricing model, the options pricing problem under regime-switching model is more complicated because it requires to handle a system of coupled partial differential equations (PDEs) for European options and partial differential complementarity problems (PDCPs) for American options, respectively. Hence, we shall be more careful when studying the numerical solutions of these options. Recently, numerical methods and theory of solutions for European and American options under regime-switching models have been studied by several researchers, readers are referred to [12–16] and the references therein. In this study, we concentrate on pricing European options under regime-switching Kou's jump-diffusion model.

European options under regime-switching Kou's jump-diffusion model can be formulated as a system of coupled parabolic PIDEs with suitable boundary and terminal conditions. Although Costabile et al. [8] proposed an analytical approximation for European call options under the two-state regime-switching Merton's jump-diffusion model, there is no exact solution for three or more states of the Markov chain under the regime-switching Merton model and two or more states of the Markov chain under the regime-switching Kou model for the time being [17]. Therefore, numerical solutions are normally sought for pricing it. In recent years, there has been much research on pricing options under regime-switching jump-diffusion models using a variety of numerical methods [8,17–32]. For instance, Lee [18] applied the implicit method with three time levels to solve the PIDEs for the prices of the European options under the regime-switching jump-diffusion model, which has the advantage not only to avoid any fixed point iteration techniques at each time step, but also to evaluate directly the prices. Florescu et al. [19] designed two iterative algorithms which handles the integral term explicitly for the European options pricing problem. Rambeerich and Pantelous [20] considered a high order finite element method for pricing European, American and Butterfly options under two-state and three-state regime-switching Merton's jump-diffusion model. Kazmi [21] presented an IMEX predictor-corrector method and Kazmi et al. [22] presented a predictor-corrector nature of the L-stable method for pricing European and American options under regime-switching jump-diffusion models, respectively. A front-fixing finite element method for pricing American options under regime-switching jump-diffusion models was investigated by Heidari and Azari in [23]. Chen et al. [17] recently established an efficient and accurate IMEX finite difference (FD) method with higher order convergence accuracy to price the European and American options under regime-switching Merton's jump-diffusion models. In space discretization, they applied compact FD schemes combined with local mesh refinement strategy near the strike price to achieve the fourth order accuracy. A high-order RBF-FD method for option pricing under regime-switching stochastic volatility models with jumps has been considered by Tour et al. in [24]. Subsequently, Tour et al. [25] proposed the spectral element method and an exponential time integration scheme to price European, digital, butterfly, American, discrete and continu-