Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 1, pp. 164-180

DOI: 10.4208/aamm.OA-2022-0060
February 2024

A Mass-Preserving Characteristic Finite Difference Method For Miscible Displacement Problem

Jiansong Zhang*, Yue Yu, Rong Qin and Zhaohui Liu

Department of Applied Mathematics, China University of Petroleum, Qingdao, Shandong 266580, China

Received 4 March 2022; Accepted (in revised version) 14 August 2022

Abstract. In this article, a new characteristic finite difference method is developed for solving miscible displacement problem in porous media. The new method combines the characteristic technique with mass-preserving interpolation, not only keeps mass balance but also is of second-order accuracy both in time and space. Numerical results are presented to confirm the convergence and the accuracy in time and space.

AMS subject classifications: 65M06, 65M25

Key words: The method of characteristics, mass-preserving, finite difference, miscible displacement problem.

1 Introduction

During recent years, the research of oil reservoir numerical simulation has been an important field in modern computational mathematics. In this field, two-phase flow displacement (water and oil) is one of the most important basic problems. In this article, we will consider to construct a new numerical method for the following incompressible miscible displacement problem in porous media, which is governed by a nonlinear coupled system of partial differential equations: the pressure is governed by an elliptic equation and the concentration is governed by a convection-diffusion equation (see [1–3]):

$$\begin{cases} \nabla\cdot\mathbf{u}=g(\mathbf{x},t), \quad \mathbf{u}=-\dfrac{\kappa(\mathbf{x})}{\mu(c)}\nabla p=-r(\mathbf{x},c)\nabla p, & (\mathbf{x},t)\in\Omega\times(0,T], \\ \phi(\mathbf{x})\dfrac{\partial c}{\partial t}+\nabla\cdot(\mathbf{u}c-D\nabla c)=\tilde{c}g(\mathbf{x},t), & (\mathbf{x},t)\in\Omega\times(0,T], \end{cases} \tag{1.1}$$

*Corresponding author.
Emails: jszhang@upc.edu.cn (J. Zhang), m18766215811@163.com (Y. Yu), S21090013@s.upc.edu.cn (R. Qin), liuzhaohui2016@163.com (Z. Liu)

where Ω is a bounded domain in R^d, $(d=1,2,3)$, $p(\mathbf{x},t)$ denotes the pressure, $\mathbf{u}$ is the Darcy velocity, $c(\mathbf{x},t)$ is the relative concentration, $\kappa(\mathbf{x})$ is the permeability of strata, $\mu(c)$ is the viscosity of the fluid mixture, $\phi(\mathbf{x})$ is the porosity of the rock, $g(\mathbf{x},t)$ is the external flow rate, which is positive if fluid is being injected, the concentration $\tilde{c}$ in the source term is the injected concentration c_ω if $g(\mathbf{x},t)\geq 0$ and is the resident concentration c if $g(\mathbf{x},t)<0$. Furthermore, a compatibility condition $\int_\Omega p d\mathbf{x}=0$, $0\leq t\leq T$ must be imposed to determine the pressure.

The pressure equation is elliptic and easily handled, but the concentration equation is parabolic and normally convection-dominated. It is well known that the standard finite difference method and Galerkin finite element method applied to the convection-dominated problems do not work well, and produce excessive numerical diffusion or nonphysical oscillation. A variety of numerical techniques were introduced to obtain better approximations, such as, Yuan etc. proposed the modified method of characteristic finite element method (MMOC) in [4–9], and Russell proposed the Eulerian-Lagrangian localized adjoint method (ELLAM) in [10]. Moreover, Yang proposed least-squares mixed finite element method in [11], and Yuan, Liang and Rui etc. proposed the characteristic finite difference schemes, the modified method of upwind with finite difference fractional steps procedures, see [12–14]. Each of the above methods has its advantages and disadvantages. Upstream-weighted method tends to introduce an excessive amount of numerical diffusion near the sharp fronts into the solution. Streamline diffusion method and least-squares mixed finite element method reduce the amount of diffusion but add a user-defined amount biased in the direction of the streamline. ELLAM conserves mass locally but it is difficult to evaluate the resulting integrals. Explicit characteristic and Godunov schemes require a CFL time-step constraint. The MMOC-Galerkin scheme has much smaller numerical diffusion than those of standard Galerkin methods, and can be used with a larger time step, with corresponding improvement in efficiency and without cost in accuracy. But it fails to keep mass balance.

In [15, 16], Liang and Fu proposed a new efficient high-order mass-conservative finite difference method for the advection-dominated transport problem. This algorithm combines the characteristic technique with the conservative interpolation technique as in [17, 18]. It does not only keep mass balance but also does well in the advection-dominated diffusion problem. And then, Fu combined block-centered finite difference method with this technique for convection domminated diffusion equations in [19]. In this article, our main purpose is to use the similar technique as in [15,16] to construct a new combined numerical scheme for incompressible miscible displacement problem. In this new algorithm, the time second-order splitting technique is used to obtain a second-order mass-preserving characteristic finite difference (MPC-FD) method for the concentration. Based on the characteristic form of the advection-diffusion equations tracking back along the characteristic curve, the integrals over the tracking cells at the previous time level are treated by the conservative interpolation distribution and the diffusion terms are approximated by averaging along the characteristics. Meanwhile, the space second-order finite difference scheme is used for the pressure and Darcy velocity.