Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 1, pp. 253-278

DOI: 10.4208/aamm.OA-2021-0186
February 2024

# Free Vibration of Stiffened Plate with Cracked Stiffeners

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Received 25 May 2021; Accepted (in revised version) 16 December 2022

**Abstract.** In this paper, a new cracked stiffener model for the stiffener with a part-through and open crack is proposed, considering the compatibility condition of displacements between the plate and the stiffener. Based on the first-order shear deformation theory, the free vibration of stiffened isotropic plates with cracked stiffeners are investigated for the first time. The description of the crack parameters is based on the continuous equivalent bending stiffness and equivalent depth of the cracked beam, and it takes into consideration of shear deformation, bending-extensional coupling vibration, and eccentricity between the stiffeners and the plate. The stiffened plates with single or multiple cracked stiffeners are formulated and discussed. The Ritz method with the modified characteristic functions is applied to demonstrate the effects of crack parameters (crack depth and location) coupling with the position and number of the cracked stiffeners on the vibration frequencies and modes of the stiffened plate. The validity and accuracy of the present solutions are verified through convergence studies and compared with the finite element results.

**AMS subject classifications**: 65M10, 78A48

**Key words**: Free vibration, stiffened plate, cracked stiffener model, crack, Ritz method.

## 1 Introduction

As an essential class of engineering structures, the stiffened plate is extensively used in various fields such as aircraft and ship structures. The vibration of stiffened plates has been investigated through various analytical and numerical techniques. As Mukherjee a-

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nd Mukhopadhyay [1] summarized in their review papers, two types of models were adopted for stiffened plates in the early literature: one is the orthotropic model in which the plate was treated as an equivalent orthotropic plate with the consideration of contribution of the stiffeners; the other is the grillage approximation model in which the stiffeners ware considered as a grid attaching to the plate. Recently, a "discrete" modeling method [2] was proposed to separately create the models of the plate and its stiffening beam. Afterwards, a complete model is formed to incorporate the displacements of the plate and the stiffener through the condition for compatibility of displacement. Many previous efforts have been devoted to the static and dynamic analysis of stiffened plates based on various methods such as the Ritz method [3–5], the finite element method [6–10], the dynamic stiffness method [11–14], the differential quadrature method [15] and the meshless method [16, 17].

When a stiffened structure is ceaselessly subjected to large irregular load or cyclic load, a fatigue crack maybe thus initiate. The existence of cracks would significantly change the dynamic characteristics of the plate [18–21]. It will be a serious issue when a crack occurs in a stiffened plate since the crack reduces the stiffness of the structure and eventually leads to an unexpected failure of the structure. Many previous studies have investigated the influence of the crack on the vibration and stability of stiffened plates. Li et al. [22–24] established an extended layerwise method and extended finite element method (XFEM) for the laminated composite plates and stiffened plates. The proposed method considered the multiple delaminations and transverse crack simultaneously. Dang and Kapania [25] analyzed the buckling of a cracked-orthogonally stiffened panel via the Ritz method in which the locally distributed trigonometric series as the admissible function of displacement was adopted. Milazzo and Oliveri [26] presented a PB-2 Rayleigh–Ritz variational approach to analysis the post-buckling behavior of cracked composite plates. This study was extended to the buckling and post-buckling of stiffened composite panels [27]. Xue et al. [28] investigated the free vibration of a stiffened plate with a side crack. The crack is through-the-thickness and starts from an arbitrary position on the edge of the flat plate. They found that the fundamental frequency decreased mildly with the crack length increase, then dropped drastically once the through-the-thickness crack broke the stiffener.

As an important component of the stiffened panel, the stiffener is expected to provide enhanced stiffness and stability to the structure with the extra advantage of lightweight. However, the effects of part-through cracks in stiffener on the vibration characteristic of stiffened plate remain unexplored. In previous research, there have been numerous studies focused on the modeling of cracks in beam structures. Swamidas et al. [29] presented a review on several main categories of the vibration models of the cracked beam. One of the earlier developed models was the local stiffness reduction method which introduced a short beam with a reduced bending stiffness to model the effect of crack [30]. Another model was the discrete spring model, which treated the cracked beam as two segments and connected both with a spring system at the crack location [31]. The stiffness of the spring was related to the depth of the crack and was determined by using the concepts of