Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 2, pp. 355-372

DOI: 10.4208/aamm.OA-2021-0148
April 2024

Natural Frequencies of Composite Lattice Structure Surrounded by Winkler–Pasternak Ambient using Galerkin Method

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Received 1 May 2021; Accepted (in revised version) 20 February 2022

Abstract. The present work contains an analytical expression and solution for free vibration problem of a composite lattice cylindrical shell surrounded by Winkler–Pasternak elastic foundation with clamped edges. The foundation is simulated using a large number of linear, homogenous shear and radial springs with variable stiffness. An integrated formula for calculation of the natural frequency of lattice structure and its foundation is derived from the equations of motion of the shell implemented by Winkler–Pasternak terms based on Fourier decomposition and Galerkin method. The fundamental frequency formula concerning the foundation elements and lattice parameters is an effective means of estimation frequency in earlier design phase and also a tool to assess the vibration analysis of composite lattice cylindrical shell surrounded by an elastic foundation in design analysis and numerical solutions. The results are verified and confirmed using finite element analysis which show a very good agreement.

AMS subject classifications: 35-11, 35E05

Key words: Free vibrations, elastic foundation Winkler–Pasternak, fundamental frequency, Galerkin's method, composite lattice cylindrical shell.

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1 Introduction

Lattice composite structures resting on/or surrounded by elastic foundations, have an outstanding roll in different fields of engineering such as aerospace, mechanics, marine and modern civil structures. Different studies on the natural frequency of beams resting on elastic foundations and analytical solutions for beams subjected to arbitrary dynamic loads have been done, but rare studies about a lattice structure on elastic Winkler–Pasternak foundation could be observed. The natural frequency of finite Timoshenko

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beams on Pasternak foundations has been analyzed with six cases of bounding conditions [1]. M. K. Ahmed has studied the natural frequencies and mode shapes of a variable thickness elastic cylindrical shell, resting on Pasternak foundation using transfer matrix [2]. D. N. Paliwal and R. Pandey studied the free vibration of an orthotropic thin cylindrical shell on a Pasternak foundation [3]. Morozov et al. obtained a solution for free vibration problem of a composite lattice shell with clamped boundary conditions [4]. They used a continuous model on which, the lattice cylinder has been replaced with a thin orthotropic shell, with the same corresponding structural stiffness. Mozorov et al. [5] and Vasiliev et al. [6, 7] have considered continuous finite element models for anisogrid composite structure, using beam, shell or solid elements. An Hou et al. examined the failure modes of both cylindrical and conical composite lattice shells. They compared the numerical results obtained by finite element analysis with experimental solutions [8]. G. Totaro analyzed the local buckling failure modes for composite anisogrid lattice cylindrical shells with a typical system of hexagonal cells. The local buckling of helical ribs is normally based on a simplified and qualitative approach, based on Ritz method. This model has been verified with the aid of finite-element analysis, demonstrating a noteworthy accuracy [9]. Xu et al. investigated the natural frequencies of composite sandwich beams with lattice truss core, by combining the Euler-Bernoulli and Timoshenko beam theories. They derived the governing partial differential equations of motion, using Hamilton's principle and obtained an analytical formulation for determining the natural frequencies [10]. Frulloni et al. studied the behavior of lattice composite hollow structures that have been subjected to an external hydrostatic pressure, using finite element modeling [11]. Jeon et al. introduced the critical stress function utilized in failure criterion, from the compression test results. The finite element analysis was used to calculate the failure load with the proposed failure criterion and also the buckling load for the full-sized cylindrical lattice structures [12]. Kim et al. applied composite lattice rectangular plates for the solar panels of a high-agility satellite. They proposed an approximate method of conducting vibration, buckling analyses of the lattice plate of the solar panel with a torsional spring using the Ritz method. This method considers the buckling as well as the vibration characteristics (natural frequencies and modes). The validity was verified by comparing the results with finite-element analysis [13]. Most recent researches have been focused on failure, design, buckling and vibrations of lattice structures due to different loads and conditions, but the influence of the foundations is not considered in most of these studies. The aim of the present study is to obtain an analytical expression for natural frequencies of lattice cylindrical shells, composed of helical and radial ribs, surrounded by an elastic Winkler–Pasternak foundation. By solving the governing equation using Galerkin method, an analytical compact expression can be obtained in order to make the possibility of tuning design parameters, to reach the design goal without consumption and utilization of cost and time for numerical modeling and modal analysis. Also, the effect of the shell's length and stiffness of shearing and radial springs on natural frequencies and stability of the cylindrical lattice shell are investigated.