Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 2, pp. 373-397

DOI: 10.4208/aamm.OA-2021-0237
April 2024

Application of 2-D GDQ Method to Analysis a Thick FG Rotating Disk with Arbitrarily Variable Thickness and Non-Uniform Boundary Conditions

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Received 20 July 2021; Accepted (in revised version) 5 April 2022

Abstract. In this paper two-dimensional differential quadrature method has been used to analyze thick Functionally Graded (FG) rotating disks with non-uniform boundary conditions and variable thickness. Material properties vary continuously along both radial and axial directions by a power law pattern. Three-dimensional solid mechanics theory is employed to formulate the axisymmetric problem as a second order system of partial differential equations. The non-uniform boundary conditions are exerted directly into the governing equations to reach the eigenvalue system of equations. Four different disk profile shapes are considered and discussed. The effect of the power law exponent is also investigated and results show that by the use of material which functionally varied along the radial and especially axial directions the stresses and strains can be controlled so the capability of the disk is increased. Comparison with other available approaches in the literature shows a good agreement here in terms of computational time, robustness and accuracy of the present method. Moreover, novel applications are shown to provide results for further studies on the same topics.

AMS subject classifications: 74G15

Key words: Thick rotating disk, FG material, 2-D GDQ, variable thickness, profile shape, non-uniform boundary condition, 2-D material gradient.

1 Introduction

A most used structural element in many rotating machineries is rotating disks, they have many practical engineering applications such as turbo generators, casting ship propellers, turbojet engines, steam and gas turbine, reciprocating and centrifugal compressors, pumps and brake disk of automobiles. In all these applications, the total stresses due to centrifugal load have important effects on their strength and safety. Thus, control and optimization of total stresses and displacements field is an important designing

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task [1]. Therefore more researches have been done on rotating disk analysis some of which are mentioned here. Afsar et al. [2] studied a thin circular FGM disk subjected to a thermo-elastic field using the finite element method. They reveal that the thermo-elastic field in FGM disks is significantly influenced by temperature distribution, the angular speed of the disk and the inner and the outer surface temperature difference. Asghari and Ghafoori [3] presented a two dimensional plate stress analysis for a three dimensional FGM rotating disk. Their results showed that although the plane-stress solution satisfies all the governing three dimensional equations of motion and boundary conditions it fails to give a compatible three dimensional strain field and it is not a valid solution. They modified the plane stress solution to reach an adequate three dimensional solution for a thick FGM rotating disk. Vullo et al. [4] presented an analytical procedure for the evaluation of elastic stresses and strains in a non-linear variable thickness rotating disk. They defined a density variation along the radial direction and a relation between the stress state and displacement field. They demonstrated that the results obtained by this method perfectly match those obtained by FEA. Nie et al. [5] analyzed the axisymmetric deformation of an isotropic rotating disk with its thickness, mass density, thermal expansion coefficient and shear modulus varying in the radial direction. They used the differential quadrature method for solving the non-homogeneous ordinary differential equations with variable coefficients for airy function. They also analyzed the challenging problem of tailoring the variation of either the shear modulus or the thermal expansion coefficient in the radial direction. Jahed et al. [6] considered an inhomogeneous disk model with variable thickness and used the variable material properties method to obtain the stress field under rotation and a steady temperature field. They modeled the rotating disk as a series rings of different but constant properties and arrived the optimum disk profile by sequentially proportioning the thickness of each ring to satisfy the stress requirements. Alexandrova et al. [7] studied the plane state of stress in an elastic-perfectly plastic isotropic rotating annular disk with constant thickness and density mounted on a rigid shaft. Hosseini et al. [8] present the stress analysis of a rotating nano-disk made of functionally graded materials with non-linearity varying thickness based on strain gradient theory. They examined the effects of various parameters such as graded index and thickness profile on stresses. Their results show that the effect of thickness parameters is greater than the effect of the graded index and the difference between the stress predicted by the classical theory and the strain gradient theory is large when the thickness of the nano-disk is small. Farshi et al. [9] considered an inhomogeneous disk with variable thickness and used the variable material properties and an optimization process to calculate the stress and optimized the disk profile.

Functionally graded materials (FGM) are composites in which the volume fraction, sizes and shapes of materials constituents can be varied continuously to get desired smooth spatial variations of macroscopic properties such as the elastic modulus, mass density, shear modulus, heat conductivity, etc. [3]. The functionally graded materials are the materials with the designing capability and are usually confirmed from metals and ceramics phase. Based on an adequate mixture rule, the volume fraction of each phases