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Calculation of Four-Dimensional Unsteady Gas Flow via Different Quadrature Schemes and Runge-Kutta 4th Order Method

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Abstract. In this study, a $(3+1)$ dimensional unstable gas flow system is applied and solved successfully via differential quadrature techniques based on various shape functions. The governing system of nonlinear four-dimensional unsteady Navier–Stokes equations of gas dynamics is reduced to the system of nonlinear ordinary differential equations using different quadrature techniques. Then, Runge-Kutta 4th order method is employed to solve the resulting system of equations. To obtain the solution of this equation, a MATLAB code is designed. The validity of these techniques is achieved by the comparison with the exact solution where the error reach to $\leq 1\times 10^{-5}$. Also, these solutions are discussed by seven various statistical analysis. Then, a parametric analysis is presented to discuss the effect of adiabatic index parameter on the velocity, pressure, and density profiles. From these computations, it is found that Discrete singular convolution based on Regularized Shannon kernels is a stable, efficient numerical technique and its strength has been appeared in this application. Also, this technique can be able to solve higher dimensional nonlinear problems in various regions of physical and numerical sciences.

AMS subject classifications: 34B05, 76N15, 35G15, 65F05, 65F10, 65F45

Key words: Statistical analysis, Runge-Kutta, discrete singular convolution, sinc, quadrature approach, gas dynamics, adiabatic index.

1 Introduction

Fluid mechanics play a significant role in this work by unsteady gas flow. Unsteady gas flow of an ideal polytropic gas in three-dimensional is a system of nonlinear partial differential equation. The dynamic flow of incompressible fluid is defined via Navier–Stokes

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(N-S) equations where the unknowns are the velocity, pressure, and density as functions of space (x,y,z) and time variables [1,2]. A result of these equations forecasts the manner of the fluid, assuming its initial and boundary conditions are known. So, these equations are considered one of the extreme important types of mathematical physics [3-5].

Recently, researchers applied different methods to get the exact and numerical results of these problems. Raja et al. [6,7] applied the Lie group of transformations technique to get the result of unsteady Euler system of gas dynamics. Also, Rashed [8] solved the previous system via an optimal equation of Lie symmetry vectors. Asymmetry analysis was applied by Fuchs and Richter [9]. Lie symmetries was developed by Murata [10] to obtain the solution of 2-D system in radial coordinates. Further, Arora et al. [11] investigated strong shocks in a non-perfect relaxing gas by the previous technique. Substitution principles were carried out by Oliveri and Speciale [12-18] to solve the unsteady equations of ideal gases and perfect magneto-gas dynamics equations. Chirkunov et al. [19] analyzed the gas dynamics with zero sound velocity. There are a lot of numerical techniques to solve N-S equations where these systems are nonlinear and complicated. Babaev et al. [20] demonstrated solution for Navier–Stokes system via Variational Iterative techniques. Numerous contributions in analysis Navier–Stokes system using Finite difference (FD) scheme have been published by various authors [21-23]. Zhao et al. [24] studied the original boundary condition-enforced immersed boundary method for simulation of incompressible flows having moving boundaries. Yuan et al. [25] examined a new gas-kinetic flux solver to simulate the compressible and incompressible flows for continuum and slip regimes by using finite volume method and Boltzmann equation. Many software programs are developed depending on Implicit Finite Difference schemes like MIKE-11 [26] and HEC-RAS [27] for solving the nonlinearity of unsteady flow equations. Zhou et al. [28] implemented the circular function-based gas-kinetic scheme to moving boundary problems for moving grids with finite volume method. A simplified and efficient multiphase lattice Boltzmann flux solver model was proposed for multiphase flows large density ratio by Yang et al. [29] The extrapolation formulation was presented to compute the compressible Navier–Stokes–Fourier equations that considers slip and jump boundary conditions by Shterev [30]. Lai [31] used method of characteristics (MOC) for solving the unsteady open-channel flow. This method depends on the choice of grid points to achieve stability, it is also complex and needs more time in programming compared to the other schemes. But the researchers investigated and developed a new numerical scheme with more convergence, stable and efficiency to raise numerical modeling abilities. Differential quadrature (DQ) scheme has been developed to solve different differential equations as linear or nonlinear. Differential quadrature (DQ) technique is a stable, converge, and efficient technique for solving various problems of fluid mechanics with small number of points and less calculations effort [32-37,40]. Shu [32,37] issued different searches for solving N–S equations and boundary condition in the field of fluid mechanics by generalized differential quadrature method (GDQM). Rosa et al [38] applied Differential quadrature technique to estimate the identification of the stiffness of structural elements. Alqahtani and Jiwari [39] studied the features of nanofluid flow