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A Two-Level Crank-Nicolson Difference Scheme and Its Richardson Extrapolation Methods for a Magneto-Thermo-Elasticity Model

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Abstract. This study is concerned with numerical solutions of a Magneto-Thermo-Elasticity (MTE) model via a combination of energy-conserving finite difference method (FDM) with Richardson extrapolation methods (REMs). Firstly, by introducing two auxiliary functions and using second-order centered FDM and Crank-Nicolson method to approximate spatial and temporal derivatives, respectively, a two-level energy-conserving FDM is established for a MTE model. The priori estimation, solvability, and convergence are derived rigorously by using the discrete energy method. Secondly, to improve computational efficiency, a class of REMs are also designed by constructing the symbolic expansion of numerical solutions. Finally, numerical results confirm the efficiency of the proposed algorithms and the exactness of the theoretical findings.

AMS subject classifications: 65M06, 65M12

Key words: Finite difference method, Richardson extrapolation methods, energy conservation, priori estimation, solvability, convergence.

1 Introduction

1.1 Mathematical model and related studies

Magneto-thermo-elasticity theories [1–21] have been fully studied in the past few decades because of its important applications in various fields, such as micro electromechanical systems, acoustics, geophysics. Traditional coupled thermo-elasticity theory, which is established based on a parabolic heat equation, admits an infinite speed for the propagation of thermal signals in elastic solids. However, this admission contradicts physical facts, especially in problems concerned with thermal shocks (cf. [1]). The generalized

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theories, which involve a hyperbolic-type heat transport equations of showing finite velocities of heat distributions, are confirmed by many experiments showing the actual occurrence of wave-type heat transport in solids (cf. [11]). Thus, the generalized thermo-elasticity theories have been proposed because they are more accurate and efficient than traditional theories in simulating practical problems. In general, there are three kinds of non-classical thermo-elasticity theories. One of non-classical thermo-elasticity theory was developed by Lord and Shulman in 1967 (cf. [3]). The authors presented a wave-form thermal equation by defining the term relaxation time and considering a new law of heat conduction instead of Fourier's law. In comparison with the classical theory, the equation of energy was only changed in Lord-Shulman theory. The second generalized theory is proposed by Green and Lindsay in 1972 (cf. [4]). The temperature rate dependent on two variables of the relaxation times was considered, and two disparate lag times in stress correlation and entropy expression were introduced. Comparing with classical theory, the equations of energy and motion were both altered in Green-Lindsay theory. The final non-classical theory was presented by Green and Naghdi (cf. [5–7]). In Green-Naghdi theory, displacement-temperature-flux rate in Fourier's law was used, and the energy of thermal stream was not wasted. Green and Naghdi provided important modifications in the constitutive equations that can describe a much wider class of heat flow problem (cf. [5–7]). To satisfy various applications, there have been growing many magneto-thermo-elasticity models with more general forms, such as phase-lag Green-Naghdi models [15–18], fractional GN II thermoelasticity models [19] and Cattaneo-type thermoelasticity [20]. Some detailed reviews about the classical and extended theories, please refer to the references [8, 9, 21] and the related references therein.

Some analytical methods (cf. [12–14, 22–24]), such as the generalized variational principle, Laplace transform method, eigenfunction expansion method and energy method, have been put forward to research the properties of analytical solutions for various thermo-elasticity models. However, it is very difficult for us to obtain the analytical solutions of the generalized models with arbitrary initial-boundary conditions. Fortunately, a lot of numerical methods, such as, numerical integration methods [25–27], finite element methods [28–31], boundary element method [32], collocation methods [33–36] and finite difference methods [37–41], have been developed by scientists and engineers for them. However, mathematical analyses including convergence, stability, posteriori estimation, and energy conservation or dissipation, have not been studied for most of them. Thus numerical results may unconvincing or unreliable. Meanwhile, these algorithms stated above are not energy-conserving methods. Moreover, most of them are three-level or more than three-level schemes. Thus, to start computation, numerical solutions at initial time levels are solved by other schemes. This increases computational cost and complexity. Worse still, to the best of our knowledge, little attention has been paid on the numerical methods for MTE models which posses high-order accuracies in both time and space. In this study, taking a G-N model for example, a Crank-Nicolson energy-preserving FDM and corresponding REMs are devised. The current methods can overcome the deficiencies stated above.