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Stability Analysis of Axially Functionally Graded Beams Using the Differential Quadrature Method

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Abstract. This paper investigates the critical buckling behavior of axially functionally graded (FG) material beams with three end support conditions. The FG materials are assumed to have continuously graded based on a power-law function of the volume fractions of the constituents. The governing equation for buckling is derived and solved using the differential quadrature (DQ) method. A comparison between the results obtained from the DQ method and the analytical approach reveals excellent agreement. The effects of various parameters, such as the gradient index and boundary conditions, on the critical buckling load is thoroughly analyzed. The findings highlight the efficiency and accuracy of the DQ method for analyzing functionally graded beams. Moreover, the insights gained from this study can inform the design and optimization of functionally graded structures.

AMS subject classifications: 74G15

Key words: Functionally graded beam, differential quadrature method, buckling, TTO model.

1 Introduction

Functionally graded (FG) materials are composite materials that exhibit unique properties due to a gradient in composition, microstructure, and properties over their volume, which cannot be achieved with traditional homogeneous materials. The advantages of FG materials such as high strength, light weight, and efficient energy absorption, making them increasingly used in various fields, including automotive, aerospace, mechanical engineering, civil engineering, and others [1,2]. Due to their wide range of applications, FG structures have attracted a great deal of attention from researchers who have investigated their mechanical behavior, including their vibration, buckling, and bending behaviors, among other challenging topics [3,5].

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Until today, there have been numerical studies addressing the buckling problem of non-homogenous beams with material properties gradient along the thickness direction [6–8]. However, only a limited number of researchers have investigated the mechanical behavior of Functionally Graded (FG) beams with varying material properties in the length direction. Owing to the varying cross-sectional area and material mechanical parameters along the beam axis, the governing equations describing the buckling and vibration of axially FG beams are differential equations with variable coefficients, making it nearly impossible to obtain closed-form solutions. Korak Sarkar obtained closed-form solutions for axially FG beams with uniform cross-section and fixed-fixed boundary conditions by employing a specific polynomial variation in the material parameters, including the material mass density and elastic modulus [9]. To simplify the problem, various numerical methods have been proposed for studying the buckling and vibration of FG beam based on different beam theory. Consequently, numerous numerical techniques and approximate methods have been developed to determine the critical buckling load of axially FG beams. These methods include the semi-inverse method [10], Rayleigh's quotient method [11], finite element method [12–14], boundary element method [15], and various other approaches [16–21].

Recently, a new numerical technique called the differential quadrature method has been proposed as an effective numerical method for solving nonlinear differential equations and was first introduced by Bellman and his coworkers [22]. Since then, it has been gained recognition as an effective approach in various scientific and engineering domains [23–26]. For example, Yang and Shen examined the dynamic response of initially stressed FG rectangular thin plates subjected to partially distributed impulsive lateral loads [27]. Lv et al. assumed that the beams are isotropic, with varied Young's modulus and utilized a hybrid state space-based DQ method to obtained the elasticity solutions for static bending and thermal deformation of 2D FG beams [28]. Based on First-order shear deformation theory, Tornabene investigated the dynamic behavior of moderately think FG conical and cylindrical shells, as well as annular plates [29]. Khalili et al. introduced a mixed method to investigate the dynamic behavior of FG beams subjected to moving loads [30]. The Rayleigh-Ritz method was employed to discretize the spatial partial derivatives, while the DQ method was used to discretize the temporal derivatives. In their study, the DQ method exhibited superior accuracy compared to the Newmark and Wilson methods, particularly when employing large time step sizes.

They employed the Rayleigh-Ritz to discretize the spatial partial derivatives and used the DQ method to discretize the temporal derivatives. Compared to the Newmark and Wilson methods, the DQ method gives better accuracy using the large time step sizes in his study. Li and Shi employed the state-space-based DQ method to investigate a multi-layered cantilever beam composed of FG piezoelectric material. They determined the non-dimensional frequency for three distinct boundary conditions, which exhibited good agreement with the finite element [31]. Nateghi et al. conducted a size dependent buckling analysis of FG micro beams using the modified couple stress theory [32]. The buckling load predicted by modified couple stress theory significantly deviated from