Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 5, pp. 1121-1151

DOI: 10.4208/aamm.OA-2023-0063
October 2024

Stability Analysis for Modelling 3D Poroelastic Wave Propagation by High-Order Staggered-Grid Schemes

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Received 28 February 2023; Accepted (in revised version) 15 March 2024

Abstract. In this paper, we investigate the numerical stability for solving the three dimensional (3D) poroelastic wave equations with the high-order staggered-grid method. First by introducing some proper difference operators, we construct the arbitrary high-order staggered-grid schemes for 3D poroelastic wave equations with spatially varying media parameters. Then the stability condition of the schemes is derived firstly. The result is an explicit restriction of time step, which only depends on the difference coefficients and the spatially varying media parameters. The condition is sufficient and can be computed prior to numerical computations, which is very helpful for us to find suitable time step and spatial grid size in numerical computations efficiently. For numerical computations, absorbing boundary conditions with perfectly matched layer (PML) based on real prolongation variables are derived. Numerical computations of 3D poroelastic wave propagation with PML are completed. The results show the effectiveness of our developed method.

AMS subject classifications: 35L05, 65M06, 65M12, 74J05

Key words: Stability, 3D poroelastic wave equations, high-order schemes, modelling, absorbing boundary conditions.

1 Introduction

Wave simulation based on wave equations has many applications in some areas such as geophysics. For example, it can be used to detect reservoirs of oil and gas [6]. Wave

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propagation in such a medium containing fluid can be described by Biot's equations or poroelastic wave equations [3,4]. Biot's linear theory for wave propagation in fluid saturated porous media is established under the following assumptions: (1) the fluid phase is continuous in a whole medium while disconnected pores are treated as part of the solid matrix; (2) the porous medium is statistically isotropic; (3) the microscopic pore size is much smaller than the seismic wavelength; (4) the deformations are small; (5) the solid matrix is elastic. Biot's theory was confirmed by Burridge and Keller [5] based on the dynamic equations which govern the behavior of medium on a microscopic scale. Plona [25] also confirmed Biot's theory through experiments. By using a model based on the poroelastic wave equations, the effects of fluid, pressure, porosity and permeability between phases can be taken into account, which provides more accurate solutions that cannot be obtained through a pure elastic or acoustic wave equation. Numerical modelling helps us to understand the wave characteristics.

There are many numerical methods developed to solve the poroelastic wave equations, including the finite-difference method [14, 24, 30, 31, 39], the finite volume method [10,35], the finite element method [2,7], the discontinuous Galerkin method [11, 16,26] and a multiscale method [36]. As the most computationally efficient method, the finite-difference (FD) method is still very popular in wave simulation. In [9], a centered-grid FD scheme based on the MacCormack FD algorithm has been developed for heterogeneous poroelastic media. In [32], a staggered-grid FD method is developed for poroelastic wave equations and the result shows that the staggered-grid scheme has a higher accuracy than the traditional second-order, centered-grid finite-difference scheme. In [33], a new hybrid finite difference/control volume method for solving the three-dimensional poroelastic wave equations in the spherical coordinate system is developed. In this paper, we apply the staggered-grid FD method to solve the 3D heterogeneous poroelastic wave equations. Furthermore, we focus on investigating the stability of the corresponding schemes.

The stability of FD schemes for poroelastic wave equations play an important role in numerical computations. It can suggest us how to choose computational parameters such as the time step and spatial grid size effectively. For the homogeneous case, some valuable results have been developed [15, 20, 23, 37]. In [20], the stability condition in one dimension is derived formally. In [15], the stability in two dimensions for the optimal implicit FD scheme is obtained. In [23], the stability is studied for the 3D rotated and standard staggered-grid schemes. In [37], a sufficient and efficient stability condition for 3D high-order staggered-grid FD schemes is developed. To our best knowledge, these are the main references on the topic of stability conditions for solving poroelastic wave equations numerically. It is worth mentioning that all these stability results are obtained for homogeneous elastic parameters and so the von Neumann method is applicable. In [38], the stability of high-order FD schemes for 2D heterogeneous poroelastic wave simulation is investigated. It is an explicit expression by material parameters and the difference scheme coefficients. As we can see, the stability analysis in the 3D heterogeneous case is still very limited. In this paper, we investigate the stability of the 3D