Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 5, pp. 1277-1296

DOI: 10.4208/aamm.OA-2024-0021
October 2024

An Efficient Regularized Lattice Boltzmann Method to Solve Consistent and Conservative Phase Field Model for Simulating Incompressible Two-Phase Flows

Xuguang Yang[1] and Yuze Zhang[2,*]

[1] *School of Mathematics and Statistics, Hunan First Normal University, Changsha, Hunan 410205, China*
[2] *School of Mathematical Sciences, Nanjing Normal University, Nanjing, Jiangsu 210046, China*

Received 2 February 2024; Accepted (in revised version) 15 May 2024

Abstract. In this study, an efficient regularized lattice Boltzmann model aimed at solving the consistent and conservative phase-field model is developed. This model is composed of the conservative Allen-Cahn equation, the momentum equation featuring a modified mass flux, and the associated consistency conditions. Consequently, two distribution functions are introduced within the framework of the regularized lattice Boltzmann model: one dedicated to the conservative Allen-Cahn equation, and the other designed for addressing the fluid dynamics equations. In order to accurately recover the momentum equation and ensure the consistency of mass and momentum transport, a simple force distribution function with a auxiliary term is incorporated into the regularized lattice Boltzmann model. To assess the capabilities of the current regularized lattice Boltzmann model, simulations of various two-phase flow problems with substantial density ratios have been conducted, including layered Poiseuille flow and spinodal decomposition. These simulations demonstrate excellent agreement with previously published numerical results. Additionally, numerical investigations into Rayleigh-Taylor instability indicate that the present regularized lattice Boltzmann model can accurately and stably track interfaces with high precision.

AMS subject classifications: 76D05, 76T10, 76P05

Key words: Lattice Boltzmann method, phase field model, two phase flow.

1 Introduction

Multiphase flows are commonly encountered in nature, biological systems, and engineering applications, often involving nonlinear, non-equilibrium, and multiscale phenomena

*Corresponding author.
Emails: xuguang1988@163.com (X. Yang), 05439@njnu.edu.cn (Y. Zhang)

[1, 2]. Simulating multiphase flows poses a challenging task within the realm of computational fluid dynamics. In the field of computational fluid dynamics, the simulation of multiphase flows is a particularly challenging task due to the intricate nature of the phenomena involved. Under conditions of high deformation rates, conventional methods may encounter numerical difficulties in accurately capturing the interfaces between different fluids, thereby elevating the risk of interface rupture.

In recent years, diffuse interface (DI) methods [3–7] have garnered considerable attention for addressing intricate challenges posed by complex multiphase flow phenomena. These methodologies have proven successful in diverse applications, such as the ascent of individual and multiple bubbles under gravity [8], the exploration of micron-scale droplet interactions [9], and the analysis of thermo-capillary flows [10]. The widespread adoption of DI methods is primarily attributed to their robust theoretical foundation rooted in phase field theory [11] and their adeptness in managing intricate morphological transformations of phase interfaces without explicit knowledge of their spatial coordinates [12–14]. Within the realm of DI approaches, the lattice Boltzmann method (LBM) has emerged as a compelling computational framework for simulating multiphase flows, nonideal gases, and flow fields with intricate geometries [15–19]. Phase segregation and interfacial dynamics, as well as employing various boundary conditions, which are difficult to deal with in the traditional methods, can easily be accounted for by the LBM via incorporation of intermolecular interactions and mesoscopic equations. Over the past few decades, numerous LBMs tailored for multiphase flow have been proposed, categorizable into four main types: the color-gradient model [20], pseudopotential model [21, 22], free-energy model [23, 24], and phase-field-based model [25–27]. Notably, the phase-field-based model, which substitutes sharp fluid/material interfaces with thin but nonzero thickness transition regions, has gained increasing prominence as a modeling choice for capturing the motion of multiphase fluids. In the traditional phase-field models, an order parameter, governed by the Cahn-Hilliard (CH) or Allen-Cahn (AC) equations, is conventionally employed for interface tracking. Given its conservative nature, the CH equation holds popularity within the two-phase flow community. Zheng et al. [28, 29] introduced a spatial difference term into the distribution function, proposing a novel LB model for CH equation recovery. In a similar vein, Zu and He [30] utilized a spatial difference term in the equilibrium distribution function to ensure correct retrieval of the CH equation. Liang et al. [31] further advanced an LB model for the CH equation, introducing a time-derivative term in the evolution equation, with the capacity to eliminate additional terms arising in the recovered equations. However, it is evident that developing LB models based on the CH phase-field model involves addressing the computational challenges posed by the fourth-order spatial derivative and precludes CH equation retrieval through second-order Chapman-Enskog analysis. An alternative phase-field model deemed suitable for two-phase flows is the conservative AC model, essentially constituting a convection-diffusion equation with a source term. In the LB method community, Geier et al. [32] devised a central-moment LB model for the local AC equation while ensuring mass conservation. Subsequently, Fakhari et al. [33] pre-