Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **10**, No. 1, pp. 138-158

DOI: 10.4208/aamm.2015.m1357
February 2018

Multiple Relaxation Time Lattice Boltzmann Simulation of 2D Natural Convection in a Square Cavity at High Rayleigh Numbers

Mohamed Mahdi Tekitek*

Mathematics Department, Faculty of Science of Tunis, University Tunis El Manar, 2029, Tunis, Tunisia

Received 24 November 2015, Accepted (in revised version) 8 May 2017

Abstract. Natural convection in a square cavity at high Rayleigh numbers is simulated by multiple relaxation time (MRT) lattice Boltzmann method (LBM) with a separate distribution function to solve the temperature. The Rayleigh numbers examined here range from $Ra=10^3$ to $Ra=10^8$. For Rayleigh numbers below 10^8, the flow remains stationary and transition occurs beyond $Ra=2\times10^8$. Unsteady results at higher Rayleigh numbers ($Ra=10^9$ and $Ra=10^{10}$) are also investigated. To the best of our knowledge, this is the first accurate study which involves the high Rayleigh numbers $Ra=10^9,10^{10}$.

AMS subject classifications: 65Z05, 76N15, 80A20, 82C20

Key words: Thermal lattice Boltzmann model, multiple relaxation time model, double population, natural convection, square cavity.

1 Introduction

Lattice Boltzmann method (LBM) [1,2] has been successfully applied to various hydrodynamic problems and the major advantage of the LBM is its explicit formulation. However, its application to non-isothermal problem is limited because of the numerical instability for thermal models [3]. In general, there are three thermal lattice Boltzmann methods (TLBM) named the multispeed approach [4], the passive scalar approach and the double population approach. The multispeed approach adopts a single distribution function in order to obtain the macroscopic dynamic and thermal equations [4]. However, this approach suffers from lack of numerical stability. The passive scalar approach also called hybrid method consists of approximating the velocity field using LBM and the macroscopic temperature employing different numerical methods (e.g., finite difference or finite volume) [5, 6]. This approach is more stable than the multispeed approach. It has,

*Corresponding author.
Email: mahdi.tekitek@fst.utm.tn (M. M. Tekitek)

however, two disadvantages. First, the viscous heat dissipation and compression work done by the pressure can not be incorporated, and second the simplicity of the LBM is lost. The double population method, was first used by He et al. [7]. This approach can be regarded as another version of the passive scalar method. In fact to solve for the macroscopic temperature another LBM distribution is used. This model has a better numerical stability than the multispeed approach, and the viscous heat dissipation and compression work done by the pressure can be solved implicitly. Peng et al. [8] proposed a simplified thermal energy distribution model where the compression work done by the pressure and the viscous heat dissipation are neglected. By introducing a forcing function, Guo et al. [9] proposed a thermal lattice BGK equation with viscous heat dissipation in the incompressible limit. The thermally driven cavity with adiabatic top and bottom walls (also called natural convection in a square cavity) is a classical benchmark to examine the accuracy of the scheme. The solution is given for 4 values of the Rayleigh number (Ra), ($Ra=10^3$, 10^4, 10^5 and 10^6). The value of the Prandtl number (Pr) is equal to 0.71, which corresponds to a cavity filled by air. The reference solution of this problem is given by De Vahl Davis [10]. To validate the double population LBM method a few researchers [11–15] have carried out the above problem. We note here that most of these works use simple relaxation time (SRT), also called Lattice Boltzmann Bhatngar-Gross-Krook (LBGK). This is due to the extreme simplicity of this method.

In this paper we present a novel double population approach using multiple relaxation time lattice Boltzmann method (MRT-LBM) with D2Q9 (i.e., the notation "D*d*Q*q*" denotes a lattice Boltzmann scheme with d space dimensions and q velocities) lattice model for solving velocity field and another D2Q9 for solving macroscopic temperature. First we validate our model by considering natural convection in a square cavity when the flow is laminar (i.e., Rayleigh number is less then 10^8). Then, we consider the cases $Ra=10^9$ and $Ra=10^{10}$ where the flow is fully turbulent by using very fine mesh. We show that our result is accurate and the closest to the benchmark result of Le Quéré [16] than previous results using the double population LBM approach [12, 13] and [17]. This paper is organized as follows. In Section 2, a brief overview of the MRT D2Q9 for the advection-diffusion problems and the MRT D2Q9 for fluid is presented. In Section 3, the thermal LBM for the simulation of a Boussinesq fluid in a square cavity is introduced. In Section 4, the results of this articles are presented and discussed. Finally, Section 5 concludes the paper.

2 Multi relaxation time Lattice Boltzmann method

2.1 Dynamic field

We consider the classical model D2Q9 with nine discrete velocities and with three conservations to model fluid problems. Let $\mathcal{L}$ be a regular lattice parametrized by a space step Δx, composed by a set $\mathcal{L}^0 \equiv \{x_j \in (\Delta x\mathbb{Z})\}$ of nodes or vertices. Δt is the time step of the evolution of LBE and $\lambda \equiv \frac{\Delta x}{\Delta t}$ is a (constant) ve-