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Numerical Comparison of the LOOCV-MFS and the MS-CTM for 2D Equations

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Abstract. The method of fundamental solutions (MFS) and the Collocation Trefftz method have been known as two highly effective boundary-type methods for solving homogeneous equations. Despite many attractive features of these two methods, they also experience different aspects of difficulty. Recent advances in the selection of source location of the MFS and the techniques in reducing the condition number of the Trefftz method have made significant improvement in the performance of these two methods which have been proven to be theoretically equivalent. In this paper we will compare the numerical performance of these two methods under various smoothness of the boundary and boundary conditions.

AMS subject classifications: 65M10, 78A48

Key words: Trefftz method, the method of fundamental solution, LOOCV, multiple scale method, non-harmonic boundary conditions.

1 Introduction

The method of fundamental solutions (MFS) [3, 9] and the Collocation Trefftz method (CTM) [14, 26, 27] are considered to be two of the most powerful boundary meshless methods for solving homogeneous equations under the condition that either the fundamental solution or the T-complete functions are available for the given differential equation. One of the great advantages of using these types of boundary-only solution procedure is simplicity. Furthermore, the solution of both of these methods converges exponentially. On the negative side, the MFS has the uncertainty of placing the source points outside the domain. Some improvements proposed like adaptive approach [6, 21] were computational expensive and nonlinear that hard to be solved, they were replaced by later fixed approach [3, 6, 8] to bear mentioned problems. Selection of optimal source

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location, accuracy of MFS primarily depending on, were dealt with by various algorithms [1,7,24,25]. In particular, the LOOCV (Leave-One-Out Cross Validation) algorithm [23] has been successfully adopted to locate the optimal source points [1]. Collocation Trefftz method is notorious for ill-conditioning of the resultant matrix. In recent years, significant advances have been developed to alleviate these deficiencies. The collocation Trefftz method was proposed to deal with the singularity by adopting nonsingular T-complete basis function [10,12]. Recently,the so-called multiple scale technique [13–15,17], a preconditioning technique, has been proposed to reduce the condition number of the resultant matrix for the nonlinear problems. In summary, both the MFS and the Trefftz method have been further enhanced and become more powerful. It is of interest to note that there is a strong mathematical connection between the MFS and the Trefftz method. Chen et al. [2] showed that the MFS and the Trefftz method are theoretically equivalent for solving the Laplace and biharmonic equations for the case of the circular domain. Liu [16] extended the equivalence between the Trefftz method and the MFS to the arbitrary domain. On the other hand, the numerical procedure of these two methods are completely different. Fu [5] has extended the MFS and collocation Trefftz method for solving nonlinear and anisotropic equations, Fu [4] has solved Laplace transformed time fractional diffusion equations which extends MFS and Trefftz Method. To the best of our knowledge, no research has been conducted to compare the performance of LOOCV-MFS and multiple scale CTM numerically. In particular, due to the recent development of the MFS in the selection of the source points and the Trefftz method in the reduction of the condition number, it is the purpose of this paper to study the performance and make a comparison of these two powerful numerical methods. The paper is organized as follows. In Section 2 we describe Laplace equations and biharmonic equations and introduce the modified Trefftz method which has multiple-scale in Trefftz bases to solve the Laplace equation with Dirichlet boundary condition and Biharmonic equation with the first and second kind, respectively. In Section 3 we briefly review the MFS for solving two kinds of equations as shown in Section 2. Numerical comparisons of these two methods have been provided in Section 4. In Section 5 we draw conclusions on the performance and comparison of these two methods.

2 The multiple-scale Trefftz method

2.1 Laplace equation

We consider the following Laplace equation with Dirichlet boundary condition

$$\Delta u(r,\theta)=0, \qquad (r,\theta)\in\Omega, \tag{2.1a}$$

$$u(x,y)=f(r,\theta), \qquad (r,\theta)\in\partial,\ \Omega, \tag{2.1b}$$

where f is a given function. In the Trefftz method, the solution in Eq. (2.1) can be approximated by the T-complete functions satisfying the governing equation Eq. (2.1a). For the