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A Study of Preconditioned Jacobian-Free Newton-Krylov Discontinuous Galerkin Method for Compressible Flows on 3D Hexahedral Grids

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Abstract. Storage requirement and computational efficiency have always been challenges for the efficient implementation of discontinuous Galerkin (DG) methods for real life applications. In this paper, a fully implicit Jacobian-Free Newton-Krylov (JFNK) method is developed in the context of DG discretizations for the three-dimensional compressible Euler and Navier-Stokes equations. Compared with the Jacobian-based methods, the Jacobian-Free approach saves the storage for the Jacobian matrix which can be of great importance for DG methods. Three types of preconditioners are investigated in which the block diagonal preconditioner requires the least storage, while the block LU-SGS and ILU0 preconditioners require more storage but are more computationally efficient. An implicit time-stepping strategy is adopted for the stability of the current solver, which is based upon a hexahedral spatial mesh and the nonlinear solver package Kinsol is used to improve the computational efficiency and robustness. Numerical results demonstrate that the preconditioned JFNK-DG solver can substantially reduce the storage requirement compared with the Jacobian based method without significantly compromising accuracy or efficiency. Furthermore, as a good compromise between efficiency and storage requirement, the ILU0 preconditioner shows the best choice of the preconditioners presented.

AMS subject classifications: 65M10, 78A48

Key words: Discontinuous Galerkin, Jacobian-free, implicit time-stepping, preconditioner.

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1 Introduction

Discontinuous Galerkin (DG) methods using high order approximations have become an attractive alternative for the solutions of systems of conservative laws [1–4]. The attractive features, such as high-order accuracy, great geometry flexibility, straightforward implementation of h/p adaptation and parallel computing make it suitable for aerodynamic applications [5–9]. The same as the classical finite element methods, DG methods can achieve high-order accuracy on grids by means of high-order polynomial approximation within elements while the physics of wave propagation is accounted for by means of solving Riemann problems at element interfaces, as in upwind finite volume methods. Despite these advantages, computational efficiency has always been a challenge in DG methods for their use in real life applications. It is noted in [15–18] that the explicit Runge-Kutta methods are not good choices for DG schemes in steady-state simulations due to their severe stability limitations. Thus in this case the use of an implicit time integration is almost mandatory. The Newton-Krylov method [10, 34] is considered as a robust and efficient approach for solving the nonlinear algebraic equations that arise from a DG discretization and has been used in [11–13]. However in order to speed up convergence, this method often combines with preconditioners, such as block diagonal (BD) [14], block Gauss Siedel (BGS) [15], Lower-Upper Symmetry Gauss Siedel (LU-SGS) factorization [16] and Incomplete Lower-Upper factorization with zero fill-in (ILU0) [17]. Nevertheless there is a very large storage requirement for the sparse Jacobian matrix and the preconditioners, especially for three-dimensional simulations, which provides a limitation on DG schemes when the grid density and/or the order of polynomial approximation increases [18]. Since only the product of the Jacobian matrix and a vector is required in the Krylov subspace methods, a difference quotient of the nonlinear function can be used as an approximation, which circumvents the construction and storage of the Jacobian matrix [19, 20]. A good preconditioning is still required in order to obtain satisfactory performance, making the schemes not completely matrix-free. Nevertheless, this can be formed based upon an approximation of the true Jacobian matrix which is easy to implement [21–23]. Regarding the specific Krylov subspace method, in this work we consider only GMRES since it is appropriate for non-symmetric and indefinite linear systems.

In this paper, a Jacobian-Free Newton-Krylov approach for the discontinuous Galerkin method is developed on hexahedral grids with a specific focus upon significantly reducing the storage requirement against the original Jacobian based solver. A novel feature of our work is that we conduct a comparative study of several preconditioners in order to investigate their computational efficiency in the framework of the JFNK-DG solver. To maximize the robustness of the three-dimensional solver, the Kinsol package [24–26] is used combining with an implicit time-stepping strategy. The developed preconditioned JFNK-DG method is used to compute a variety of flow problems on hexahedral grids to demonstrate its accuracy, efficiency and robustness. Numerical results demonstrate that the preconditioned Jacobian-Free Newton-Krylov approach works well with DG method