

## Fitted Finite Volume Method of Three Transboundary Pollution in Three Gorges Reservoir Area of Chongqing City with Emission Permits Trading by Cooperative Stochastic Differential Game

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Received 29 March 2017; Accepted (in revised version) 3 October 2017

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**Abstract.** In this paper, we present a stochastic differential game to model the three transboundary industrial pollution problems in Three Gorges Reservoir Area of Chongqing City with emission permits trading. The process of emission permits price is assumed to be stochastic and to follow a geometric Brownian motion. We make use of stochastic optimal control theory to derive the system of Hamilton-Jacobi-Bellman equations satisfied by the value functions for the cooperative games, and then propose a fitted finite volume method to solve it. The efficiency and the usefulness of this method are illustrated by an empirical study for Wanzhou District, Kaizhou District, and Yunyang County in Chongqing City.

**AMS subject classifications:** 49J20, 65N30

**Key words:** Three transboundary pollution in Three Gorges Reservoir Area of Chongqing City, stochastic differential game, emission permits trading, Hamilton-Jacobi-Bellman equation, fitted finite volume method.

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## 1 Introduction

Three Gorges Project, as one of the biggest hydropower-complex projects in the world, serves as the key project for improvement and development of Yangtze River. It has brought huge economic and social benefits, while also ecological environmental safety issues. As is known to all, the self-purification capacity of static water is much lower than flowing water, the Three Gorges Reservoir water environment is gradually deteriorating and the water environment security has attracted widespread attention.

The safety of water environment is related to the Three Gorges Reservoir Area, including Wanzhou District, Kaizhou District, and Yunyang County in Chongqing City, the success of the Yangtze River region water security, the smooth implementation of the water diversion project, and even the overall sustainable development of China. Recently, the highest impounded water level of Three Gorges Reservoir Area has been 175m, and the water flow tends to slow down, the self-purity capacity is decreasing, and the pressure of environmental pollution is increasing.

America and Canada concluded a bargain to deal with the transboundary air pollution, the transboundary air pollution meant which originated at one location and damaged another region's air quality after a traveling of the pollution. In recent years, transboundary pollution problem has received considerable attention in [7]. The differential game can be regarded as an effective instrument to deal with pollution control problems and to examine the reciprocal actions between the dynamic processes of pollution and participants' behaviors. Differential games are often used to model and analyze the actions in the case of dynamic systems. There are many players with their own goals in the system and the dynamics of players' states are modeled by a series of differential equations. In a transboundary pollution control problem, the neighboring regions or countries can be seen as the players and they aims at maximizing the joint or their own net present profits under the cooperative and noncooperative games, respectively. In recent years, many researches have been done many works to adapt climate change from



Figure 1: The map of China on the left hand side and the map of Wanzhou District, Kaizhou District and Yunyang County on the right hand side.