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Analysis and Application of Stochastic Collocation Methods for Maxwell's Equations with Random Inputs

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Abstract. In this paper we develop and analyze the stochastic collocation method for solving the time-dependent Maxwell's equations with random coefficients and subject to random initial conditions. We provide a rigorous regularity analysis of the solution with respect to the random variables. To our best knowledge, this is the first theoretical results derived for the standard Maxwell's equations with random inputs. The rate of convergence is proved depending on the regularity of the solution. Numerical results are presented to confirm the theoretical analysis.

AMS subject classifications: 65M10, 78A48

Key words: Maxwell's equations, random permittivity and permeability, stochastic collocation methods, uncertainty quantification.

1 Introduction

Uncertainty is ubiquitous in many complex physical systems, such as wave, sound and heat propagation through random media, and flow and propagation driven by stochastic forces and stochastic initial conditions. Stochastic partial differential equations (SPDEs) have played an important role in the study of uncertainty quantification (UQ) in many branches of science and engineering. In electromagnetics, the fluctuations in the producing process (such as during the lithography) of electromagnetic materials allow us to treat the permittivity and permeability as uncertain parameters (e.g., [4,7]). Stochastic Maxwell equations with additive noise were investigated in [13,14]. Due to the high dimensionality of stochastic solutions, it is very challenging to efficiently solve PDEs with uncertain parameters, and has attracted a great attention in recent years (e.g., [8,18,23], review articles [12,20] and books [10,16,26,28]).

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Due to the low convergence rate of the traditional Monte Carlo method, the stochastic Galerkin methods [2, 10, 11] have been developed and show faster convergence rates with increasing order of expansions, provided that the solutions of differential equations are sufficiently smooth in the random space. However, the system of equations resulting from the stochastic Galerkin methods is often coupled and quite expensive to solve especially for problems requiring high-dimensional random spaces. In [26], Xiu and Hesthaven proposed a class of stochastic collocation methods by taking advantage of the strength of Monte Carlo methods and the stochastic Galerkin methods. Their stochastic collocation method achieves fast convergence when the solutions are sufficiently smooth in the random space. More importantly, the stochastic collocation method is simple in implementation and the system of resulting equations is decoupled and hence is efficient to solve. The stochastic collocation methods have been applied to solve various problems. For example, Babuska et al. [1] proposed and analyzed a stochastic collocation method to solve elliptic problems with random coefficients and forcing terms. Zhang and Gunzburger [27] presented a detailed error analysis of a stochastic collocation method for linear parabolic equations with random coefficients, forcing terms, and initial conditions. Motamed et al. [17] proposed and analyzed a stochastic collocation method to solve the elastic wave equation with random coefficients. Tang and Zhou [22] proposed some rigorous regularity analysis for the random transport equation with a random wave speed and demonstrated the convergence of the stochastic collocation methods. As for Maxwell's equations, in 2006, Chauviere et al. [7] solved the time-dependent Maxwell's equations by using both the stochastic Galerkin method and stochastic collocation method. Detailed comparisons of both methods are made for uncertainties caused by physical materials, by the source wave and by the physical domain. In 2015, Benner and Schneider [4] described several techniques for uncertainty quantification for the time-harmonic Maxwell's equations by using stochastic collocation method. In our recent work [15], we analyzed a stochastic collocation method for the metamaterial Maxwell's equations with random input data.

Though stochastic collocation method has been applied to solve Maxwell's equations with uncertain parameters [4, 7], it would be interesting to develop a theory that offers insight to how uncertainty propagates through the dynamical systems and what regularity we can expect as Chauviere et al. pointed out [7, pp. 774]. One of the main purposes of this paper is to fill this gap.

The rest of the paper is organized as follows. In Section 2, we first present detailed regularity analysis of the time-dependent Maxwell's equations with random permittivity and permeability, then we establish the convergence rate for the stochastic collocation method applied to the Maxwell's equations. Numerical results are presented in Section 3 to support our theoretical analysis. Section 4 concludes the paper.

2 Maxwell's equations with random inputs