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Jacobi Spectral Collocation Method Based on Lagrange Interpolation Polynomials for Solving Nonlinear Fractional Integro-Differential Equations

Xingfa Yang[1], Yin Yang[2,*], Yanping Chen[3] and Jie Liu[4]

[1] *State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha 410082, Hunan, China*
[2] *Hunan Key Laboratory for Computation and Simulation in Science and Engineering, Key Laboratory of Intelligent Computing & Information Processing of Ministry of Education, School of Mathematics and Computational Science, Xiangtan University, Xiangtan 411105, Hunan, China*
[3] *School of Mathematical Sciences, South China Normal University, Guangzhou 510631, Guangdong, China*
[4] *Center for Research on Leading Technology of Special Equipment, School of Mechanical and Electric Engineering, Guangzhou University, Guangzhou 510006, Guangdong, China*

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Abstract. In this paper, we study a class of nonlinear fractional integro-differential equations, the fractional derivative is described in the Caputo sense. Using the properties of the Caputo derivative, we convert the fractional integro-differential equations into equivalent integral-differential equations of Volterra type with singular kernel, then we propose and analyze a spectral Jacobi-collocation approximation for nonlinear integro-differential equations of Volterra type. We provide a rigorous error analysis for the spectral methods, which shows that both the errors of approximate solutions and the errors of approximate fractional derivatives of the solutions decay exponentially in L^∞-norm and weighted L^2-norm.

AMS subject classifications: 65R20, 45J05, 65N12

Key words: Spectral method, nonlinear, fractional derivative, Volterra integro-differential equations, Caputo derivative.

*Corresponding author.
Emails: yxfhnu@aliyun.com (X. F. Yang), yangyinxtu@xtu.edu.cn (Y. Yang), yanpingchen@scnu.edu.cn (Y. P. Chen), jliu@gzhu.edu.cn (J. Liu)

1 Introduction

During the past three decades, the subject of fractional calculus (that is, calculus of integrals and derivatives of arbitrary order) has gained considerable popularity and importance, mainly due to its demonstrated applications in numerous diverse and widespread fields in science and engineering. For example, fractional calculus has been successfully applied to problems in system biology, physics, chemistry and biochemistry, hydrology, medicine and finance. In many cases these new fractional-order models are more adequate than the previously used integer-order models, because fractional derivatives and integrals enable the description of the memory and hereditary properties inherent in various materials and processes that are governed by anomalous diffusion. Hence, there is a growing need to find the solution behaviour of these fractional differential equations. However, the analytic solutions of most fractional differential equations generally cannot be obtained. As a consequence, approximate and numerical techniques are playing an important role in identifying the solution behaviour of such fractional equations and exploring their applications.

In this article, we are concerned with the numerical study of the following nonlinear fractional integro-differential equation:

$$D^{\gamma}y(t)=\widehat{f}(t,y(t))+\int_0^t \widehat{K}(t,\tau,y(\tau))d\tau+\widehat{g}(t), \quad 0<\gamma<1, \quad t\in[0,T], \tag{1.1a}$$

$$y(0)=y_0, \tag{1.1b}$$

where $0<\gamma<1$, $\widehat{f}:[0,T]\times\mathbb{R}\to\mathbb{R}$, kernel function $\widehat{K}:S\times\mathbb{R}\to\mathbb{R}$ (where $S:=\{(t,\tau):0\le\tau\le t\le T\}$) and $\widehat{g}(t):[0,T]\to\mathbb{R}$ are known, $y(t)$ is the unknown function to be determined. D^{γ} denotes fractional derivative of fractional order γ defined as Caputo derivative.

It will always be assumed that problem (1.1) possesses a unique solution, namely, the given functions $\widehat{f}(t,y)$, $\widehat{K}(t,\tau,y)$ and $\widehat{g}(t)$ will be subject to the conditions that $\widehat{g}\in C[0,T]$, $\widehat{f}$ is continuous for all x and all u and satisfies the (uniform) Lipschitz conditions:

$$|\widehat{f}(t,y_1)-\widehat{f}(t,y_2)|\le M|y_1-y_2|, \tag{1.2}$$

$\widehat{K}$ is continuous for all S, $\widehat{K}\in H^m$ for y and satisfies the Lipschitz conditions:

$$|\widehat{K}(t,\tau,y_1)-\widehat{K}(t,\tau,y_2)|\le L_0|y_1-y_2|, \tag{1.3a}$$

$$\Big|\frac{\partial^i\widehat{K}(t,\tau,y_1)}{\partial y^i}-\frac{\partial^i\widehat{K}(t,\tau,y_2)}{\partial y^i}\Big|\le L_i|y_1-y_2|, \quad i=1,\cdots,m, \tag{1.3b}$$

for all $t\in[0,T]$, $(t,\tau)\in S$ and y_1, $y_2\in\mathbb{R}$, with Lipschitz constants M and L_i being independent of y_1 and y_2.

Let $\Gamma(\cdot)$ denote the Gamma function. For any positive integer n and $n-1<\gamma<n$, the Caputo derivative, Riemann-Liouville derivative and fractional integral of order γ are respectively defined as: