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A Regularized Singular Boundary Method for Inverse Cauchy Problem in Three-Dimensional Elastostatics

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Abstract. The application of the singular boundary method (SBM), a relatively new meshless boundary collocation method, to the inverse Cauchy problem in three-dimensional (3D) linear elasticity is investigated. The SBM involves a coupling between the non-singular boundary element method (BEM) and the method of fundamental solutions (MFS). The main idea is to fully inherit the dimensionality advantages of the BEM and the meshless and integration-free attributes of the MFS. Due to the boundary-only discretizations and its semi-analytical nature, the method can be viewed as an ideal candidate for the solution of inverse problems. The resulting ill-conditioned algebraic equations is regularized here by employing the first-order Tikhonov regularization technique, while the optimal regularization parameter is determined by the *L*-curve criterion. Numerical results with both smooth and piecewise smooth geometries show that accurate and stable solution can be obtained with a comparatively large level of noise added into the input data.

AMS subject classifications: 62P30, 65M32, 65K05

Key words: Meshless method, singular boundary method, method of fundamental solutions, elastostatics, inverse problem.

1 Introduction

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Over the past two decades, some considerable effort was devoted to proposing novel computational algorithms that circumvent or greatly eliminate the problems associated with the domain and/or boundary meshing. This led to the development of various meshless or meshfree methods [1–4]. These methods, generally, can be divided into the domain-type or boundary-type schemes, depending on whether their basis functions satisfy the governing equation of interest [5–8]. For an overview of the state of the art, we refer interested readers to articles [9–14] for existing theoretical results, different algorithms, and software packages.

The singular boundary method (SBM) [15–21] is a relatively new boundary-type meshless method for the numerical solution of boundary value problems governed by certain partial differential equations. The method involves a coupling between the non-singular boundary element method (BEM) [22, 23] and the method of fundamental solutions (MFS) [7, 9, 24–29]. The key idea of the method is to fully inherit the dimensionality and stability advantages of the BEM and the meshless and integration-free attributes of the MFS. The method keeps the advantage of simplicity of the MFS and meanwhile sidesteps the troublesome fictitious boundary issue associated with the later. The key idea of the SBM was proposed in the early 2000s by Chen and Wang [30] and were later essentially improved and extended by many other authors [31–33]. Prior to this study, this method has been successfully tried for two-dimensional (2D) problems in potential and elasticity theories [15], acoustic radiation and scattering problems [18], inverse heat conduction problems [34], thin-walled structural problems [35], as well as large-scale modelling for 3D heat conduction problems [36].

Motivated by the rapidly growing interest in the area, this paper documents the first attempt to extend the SBM for the solution of inverse Cauchy problems arising in 3D elastostatics. In inverse problems, one or more of the data describing the direct problem is missing. To fully determine the process, additional data must be supplied, either other boundary conditions on the same accessible part of boundary or measurements at some internal points in the domain [37]. The inverse problems are, generally, difficult to solve numerically due to the fact that they are ill-posed in the sense that small errors in measured data may lead arbitrarily large changes in the numerical solution [38–40]. The resulting ill-conditioned algebraic equations are regularized here by employing the first-order Tikhonov regularization technique [7], while the optimal regularization parameter is determined by the L-curve criterion [41]. It is shown that the SBM can be viewed as an ideal candidate for the solution of inverse Cauchy problems, due to the boundary-only discretizations and its semi-analytical nature.

A brief outline of the rest of the paper is as follows. In Section 2, the mathematical formulation for 3D elasticity problems is briefly introduced. The methodology of the SBM and its numerical implementation for 3D elastostatics are reviewed in Section 3. The Tikhonov regularization method with the choice of the regularization parameter given by the L-curve criterion is presented in Section 4. Next in Section 5, three benchmark numerical examples involving both smooth and piecewise smooth geometries are investigated. Finally, some conclusions and remarks are provided in Section 6.