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Towards an Accurate and Robust Roe-Type Scheme for All Mach Number Flows

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Abstract. We propose an accurate and robust Roe-type scheme applied to the compressible Euler system at all Mach numbers. To study the occurrence of unstable modes during the shock wave computation, a shock instability analysis of several Roe-type schemes is carried out. This analysis approach allows to propose a simple and effective modification to eliminate shock instability of the Roe method for hypersonic flows. A desirable feature of this modification is that it does not resort to any additional numerical dissipation on linear degenerate waves to suppress the shock instability. With an all Mach correction strategy, the modified Roe-type scheme is further extended to solve flow problems at all Mach numbers. Numerical results that are obtained for various test cases indicate that the new scheme has a good performance in terms of accuracy and robustness.

AMS subject classifications: 35L65, 65M08, 76M12, 76L05

Key words: Roe scheme, low Mach number, numerical shock instability.

1 Introduction

Due to their physical background, Godunov-type schemes become one of the most effective techniques for shock-capturing. They play a fundamental role in modern CFD methods for compressible flows at moderate Mach numbers. However, it has been well demonstrated that Godunov-type schemes are not always reliable in cases where the flow speed is hypersonic or the Mach number tends to zero. The high speed flow problems usually involve complex flow phenomena, such as strong shock waves, shock-vortex interactions, shock-boundary layer interactions and shear layers. Prediction of these problems requires robust, efficient and accurate numerical methods. Unfortunately, Godunov-type schemes that involve minimal dissipation on linear degenerated waves

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are vulnerable to shock instability problems, including the carbuncle phenomenon [1]. In [2], the authors present a wide-ranging survey of numerical experiments on a large collection of flux functions for hypersonic flow computations, the results have demonstrated that none of the tested schemes is able to balance the low dissipation and the high robustness well at the same time, all the tested solvers exhibit numerical shock instabilities to some extent. The reliability of these methods will become even worse for hypersonic flow computations in cases where the computational grid is complex [3] or the schemes are extended to their higher order versions [4]. Meanwhile, it is common that compressible and weakly compressible flow regimes coexist in the domain of interest. Even in high speed flows, there are still low Mach number regions such as boundary layers, stagnation regions and the wake behind bodies. Special attention should also be paid to the accuracy of numerical methods at low speeds, for example, the aerodynamic heating predictions [5, 6] and the turbulence flow simulations [7]. This explains the need for solvers that are able to deal with problems where the flow regime may vary from low to high Mach values.

The Roe scheme [8] is perhaps one of the most famous approximate Riemann solvers. Due to its good performance for moderate Mach number flows, the Roe scheme has been widely studied and applied to simulations of engineering problems. However, the original Roe method has several shortcomings, for example, the violation of the entropy condition and non-positivity in low density flows. These deficiencies have been fixed properly [9–11], but there is still a challenging problem that has not been well explored or clearly understood. As other low-diffusion flux functions, the Roe approach also suffers from shock anomalies at strong shock cases such as the carbuncle phenomenon, which significantly reduces its reliability in hypersonic flow computations. Since its first discovery by Peery and Imaly [12], the carbuncle phenomenon has been widely studied by researchers and engineers. A large variety of work has been devoted to understanding and curing this problem. Readers are suggested to refer to [1, 13–15] and the references therein for detailed reviews of this problem. We remark that there is still no a clear consensus on the mechanism of the carbuncle phenomenon. Quirk [16] may be the first to systematically analyze the unfavorable pathologies of Godunov-type schemes for flows at high Mach numbers. He finds that dissipative numerical fluxes, which smear entropy waves and shear waves severely, are endowed with high resistance against shock anomalies. To cure the shock instability of the Roe scheme, he suggests to combine it with a more dissipative HLLE scheme [17]. Following Quirk, many works are devoted to improving the robustness of the Roe method by introducing additional numerical or physical dissipation. These modifications include the hybrid technique which combines two different schemes with different dissipative properties [16, 18], the entropy fix technique that includes additional dissipation via modifying eigenvalues of the related dissipative matrix [19–21] and the rotated Riemann solver approach that introduces more numerical dissipation by a rotated manner [22–24]. These methods are all effective in suppressing shock instabilities of numerical methods at strong shocks. However, although these modifications are implemented by different strategies, they usually rely on certain switching