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Influence of Magnetic Force on the Flow Stability in a Rectangular Duct

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Abstract. The stability of the flow under the magnetic force is one of the classical problems in fluid mechanics. In this paper, the flow in a rectangular duct with different Hartmann (Ha) number is simulated. The finite volume method and the SIMPLE algorithm are used to solve a system of equations and the energy gradient theory is then used to study the (associated) stability of magnetohydrodynamics (MHD). According to the energy gradient theory, K represents the ratio of energy gradient in transverse direction and the energy loss due to viscosity in streamline direction. Position with large K will lose its stability earlier than that with small K. The flow stability of MHD flow for different Hartmann (Ha) number, from $Ha=1$ to 40, at the fixed Reynolds number, $Re=190$ are investigated. The simulation is validated firstly against the simulation in literature. The results show that, with the increasing Ha number, the centerline velocity of the rectangular duct with MHD flow decreases and the absolute value of the gradient of total mechanical energy along the streamwise direction increases. The maximum of K appears near the wall in both coordinate axis of the duct. According to the energy gradient theory, this position of the maximum of K would initiate flow instability (if any) than the other positions. The higher the Hartmann number is, the smaller the K value becomes, which means that the fluid becomes more stable in the presence of higher magnetic force. As the Hartmann number increases, the K value in the parallel layer decreases more significantly than in the Hartmann layer. The most dangerous position of instability tends to migrate towards wall of the duct as the Hartmann number increases. Thus, with the energy gradient theory, the stability or instability in the rectangular duct can be controlled by modulating the magnetic force.

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1 Introduction

The instabilities in magnetohydrodynamics (MHD) flows have obtained much concern since it would affect the product quality of magnetic casting, stirring, and metallurgy [1]. The duct flow of an electrically conducting fluid with an imposed constant magnetic force is nearly optimal for analyzing fundamental properties of turbulence in liquid metal MHD as well as implications for many technological processes. Other applications include the continuous casting of steel or self-cooled liquid metal blankets for nuclear fusion reactors. Despite the simple geometrical setup, the flow presents several key effects: turbulence with mean shear and the Hartmann boundary layers at the walls, respectively, perpendicular and parallel to the magnetic force.

Hartmann and Lazarus [2] experimented on the flow of mercury in a homogeneous magnetic force with various aspect ratios of rectangular ducts, and investigated the changes in the skin friction and the suppression of turbulence caused by magnetic force. Brouillette and Lykoudis [3] carried out experiment on the MHD turbulent flow in a rectangular duct $5:1$ aspect ratio with insulated walls, and investigated the laminarization effect under a uniform and strong magnetic force. The skin friction coefficient C_f is observed to be a function of Hr $(\equiv Ha/Re\times 10)$.

Gradner and Lykoudis [4] studied turbulent pipe flow in a transverse magnetic force and found the reduction of the skin friction coefficient was remarkably observed at around $Hr=30$ $(R=333)$ ($R=Re/Ha$ instead of Hr). Sajid et al. [5] investigated the non-similar analytic solution for MHD flow and heat transfer in a third-order fluid over a stretching sheet. It is found that the skin friction coefficient decreases as the magnetic parameter increases or the third grade parameter increases. Ishak et al. [6] worked on the mixed convection boundary layer in the stagnation-point flow of an incompressible viscous fluid over a stretching vertical sheet. Kobayashi [7] performed the large eddy simulation (LES) of the MHD turbulent channel flow employing the Smagorinsky model (SM), the Dynamic Smagorinsky model (DSM) and the Coherent Structure model (CSM). It shows that the CSM is able to predict higher transition Hartmann number much better than the DSM. Kobayashi [8] did LES study on turbulent MHD duct flows with a uniform magnetic field perpendicular to insulated walls. It was found that the coherent structures near the Hartmann layer are suppressed more than those near the sidewall with the increasing magnetic effect. Grandet et al. [9], Takhar and Ram [10], and Duwairi and Damseh [11] studied the hydromagnetic flow in MHD. Pantokratoras [12] also investigated the MHD boundary layer flow over a heated stretching sheet with variable viscosity, and good results have been achieved.

Tinsober [13] made use of the electromagnetic force generated by the interaction electrodes and magnetic poles to make clear that the electromagnetic force is conducive to