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Parametric Vibration Analysis of Pipes Conveying Fluid by Nonlinear Normal Modes and a Numerical Iterative Approach

Feng Liang[1,2,*], Xiaodong Yang[2,*], Wei Zhang[2], Yingjing Qian[2] and Roderick V. N. Melnik[3,4]

[1] *College of Mechanical Engineering, Yangzhou University, Yangzhou 225127, Jiangsu, China*
[2] *Beijing Key Laboratory of Nonlinear Vibrations and Strength of Mechanical Engineering, College of Mechanical Engineering and Applied Electronics, Beijing University of Technology, Beijing 100124, China*
[3] *The MS2Discovery Interdisciplinary Research Institute, M2NeT Laboratory, Wilfrid Laurier University, 75 University Avenue West, Waterloo, ON, Canada, N2L 3C5*
[4] *BCAM–Basque Center for Applied Mathematics, E48009 Bilbao, Spain*

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Abstract. Nonlinear normal modes and a numerical iterative approach are applied to study the parametric vibrations of pipes conveying pulsating fluid as an example of gyroscopic continua. The nonlinear non-autonomous governing equations are transformed into a set of pseudo-autonomous ones by employing the harmonic balance method. The nonlinear normal modes are constructed by the invariant manifold method on the state space and a numerical iterative approach is adopted to obtain numerical solutions, in which two types of initial conditions for the modal coefficients are employed. The results show that both initial conditions can lead to fast convergence. The frequency-amplitude responses with some modal motions in phase space are obtained by the present iterative method. Quadrature phase difference and traveling waves are found in the time-domain complex modal analysis.

AMS subject classifications: 70K28, 70K75, 39B12

Key words: Gyroscopic continua, pipes conveying pulsating fluid, parametric vibration, nonlinear normal modes, iterative approach.

1 Introduction

The gyroscopic device is a kind of basic engineering structure with extensive applications in aerospace, navigation, petroleum and mechanical automation. The gyroscopic systems

*Corresponding author.
Email: jxdyang@163.com (X. D. Yang)

can generally be classified into two categories: one is the translating materials, such as pipes conveying fluid and axially moving systems, the gyroscopic coupling of which is caused by the coupling between general coordinates of arbitrary transverse motion; the other is rotating bodies, the gyroscopic coupling of which is due to the coupling between two directions in the motion plane [1].

In whichever of the above gyroscopic systems, the presence of skew-symmetric gyroscopic operator in the governing equations limits analytical results but enriches dynamic behaviors dramatically, which has attracted much research attention to this field over years. Investigations of the gyroscopic systems were originated from the study of dynamics of the band saws [2]. Earlier research was confined to the analysis of natural frequencies, critical speed, and stability of the linear free vibrations [3]. With deeper understanding of the gyroscopic dynamics, the nonlinear properties of gyroscopic systems gradually became research focus, including the responses to external excitation and parametric resonance studied by perturbation method [4] and numerical method [5], and the mode interactions due to internal resonance [6]. Currently, great progress has been made towards exploration of various gyroscopic structures, involving the fluid-structure interaction systems [7,8], axially moving systems [9] and rotating bodies [10]. The three-body problems in celestial mechanics, a discrete gyroscopic system, are also concerned in nonlinear dynamic realm [11].

Natural frequencies and vibration responses of a non-gyroscopic system are often predicted by means of modal analysis, even in the case of mode interactions. However for a gyroscopic system, the modal analysis becomes complicated because the complex modes must be involved to capture the dynamics in nature [12, 13]. In the classical real modal analysis, as we have known, if the coordinate/velocity of one arbitrary DOF is given, those of all the other DOFs can be represented as functions of the given coordinate/velocity. It implies that the coordinates/velocities of all DOFs will hold the same phase or the phase difference of π, which leads to an in-unison vibration. Whereas in a complex modal analysis, the coordinate/velocity of each DOF is the function of combination of the given coordinate and velocity. There thus exist any possible phase differences among the DOFs and an out-of-unison vibration is present. Current researches on complex modes are mostly regarding their applications in linear damped systems since the fundamental significance of modal analysis lies in the design of mechanical systems in linear regimes, such as the acquisition of natural frequencies and mode shapes of a dynamic structure [14, 15]. Related studies have been extended to the rotor systems [16] and biomechanics [17]. Whereas in many cases, the nonlinear effects on these mechanical structures are often hard to be neglected, such that the subsequent concentrations have been put into the nonlinear modes. Among the related researches, the contribution of Nayfeh [18] is respected as a significant foundation in this area, wherein comprehensive nonlinear interactions, involving nonlinear mode couplings, have been explored and summarized by analytical, numerical and experimental approaches. With these developments, nonlinear complex modes are gradually adopted to extend the conventional linear complex modal theory into nonlinear fields, which shed a new light on the modal analy-