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Partitioned Time Stepping Method for Fully Evolutionary Navier-Stokes/ Darcy Flow with BJS Interface Conditions

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Abstract. In this report, a partitioned time stepping algorithm for Navier-Stokes/Darcy model is analyzed. This method requires only solve one, uncoupled Navier-Stokes and Darcy problems in two different sub-domains respectively per time step. On the interface, the simplified Beavers-Joseph-Saffman conditions are imposed with an additional assumption $\mathbf{u}\cdot\mathbf{n}_f > 0$ (not hold for general case but still in many situation, such as the gentle river). Under a modest time step restriction of the form $\Delta t \le C$, where $C = C$ (physical parameters), we prove stability of the method and get the error estimates. Numerical tests illustrate the validity of the theoretical results.

AMS subject classifications: 65M55, 65M70

Key words: Fully evolutionary Navier-Stokes/ Darcy problem, partitioned time stepping method, Beavers-Joseph-Saffman, interface conditions, error estimate.

1 Introduction

The transport of substances coupling between surface water and groundwater is an important problem of great current interest. Generally speaking, surface water are governed by the Navier-Stokes equation (or Stokes equation) and for groundwater, one popular choice is the Darcy law. The two flows are coupled as a coupled model through certain interface conditions.

There have been a lots of studies of the numerical solutions of the coupled Stokes-Darcy equations, see [1–13]. To improve the efficiency, on the numerical analysis of

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the fully evolutionary Stokes-Darcy problem with the Beavers-Joseph-Saffman condition, Mu and Zhu [8] study a partitioned method. Li et al. [10] present a decoupling method with different subdomain time steps. Cao, Gunzburger et al. [7] study a fully, monolithically coupled implicit method for the much harder and physically more accurate case of Beavers-Joseph coupling condition (without Saffman's simplication). Shan and Zheng [11] have proposed the partitioned time stepping method on this case.

Recently, some coupled finite element methods have been studied for solving the non-stationary Navier-Stokes/Darcy problems in [14–19]. Since the advantages of the decoupling methods, many researchers focus on the development of different decoupling methods, such as the modified characteristics finite element method in [20] and the domain decomposition method in [21].

However, to overcome the difficulties in Navier-Stokes/Darcy problems, in the above work, usually, adding the inertial effects in the balance of forces at the interface, the authors simplify the analysis by using a stronger control on the nonlinear convection term. However, we still do not figure out the physical justification of this model, although it is meaningful from a mathematical point of view.

In this paper, we mainly studied the gentle river, in which, the water infiltration satisfy the assumption: on the interface $\mathbf{u}\cdot\mathbf{n}_f > 0$. The assumption may not hold for general case, but still stand for many situation, like the gentle river, We consider it is a special case of Navier-Stokes/Darcy Model, also can be find in [22,23].

In this report, we propose a partitioned time stepping method for fully evolutionary Navier-Stokes-Darcy problem with the classical empirical Beavers-Joseph-Saffman interface condition. This method requires only solve one, uncoupled Navier-Stokes and Darcy sub-physics and sub-domain solve per time step. Most importantly, both subdomain solvers are used as a black box, each time step involves passing information across the interface followed by solving the individual subproblems independently.

The organization of the paper is as follows: in Section 2, we provide the formulation and the coupled method for the fully evolutionary Navier-Stokes-Darcy system. We present the partitioned sheme and analyze its stability and the error estimations for velocity and pressure approximation in Section 3. Numerical tests are presented to verify the theoretical results in Section 4, followed by conclusion in Seciton 5.

2 Mixed Navier-Stokes/ Darcy model

2.1 Formulation of the problem

Specifically, let us consider a surface water-groundwater system consisting of two domain, the surface water in the domain Ω_f and the groundwater domain Ω_p, the full bounded domain $\Omega\subset R^d(d=2 \text{ or } 3)$, see the Fig. 1, where $\Omega_f\cap\Omega_p=\varnothing$, $\overline{\Omega}_f\cup\overline{\Omega}_p=\overline{\Omega}$, $\overline{\Omega}_f\cap\overline{\Omega}_p=\Gamma$, Ω_f and Ω_p both touch $\partial\Omega$, see Fig 1. The flow in Ω_f is incompressible and