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Convergence Analysis of Legendre-Collocation Spectral Methods for Second Order Volterra Integro-Differential Equation with Delay

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Abstract. In this paper, a Legendre-collocation spectral method is developed for the second order Volterra integro-differential equation with delay. We provide a rigorous error analysis for the proposed method. The spectral rate of convergence for the proposed method is established in both L^2-norm and L^∞-norm. In the end numerical experiment is illustrated to confirm the theoretical analysis.

AMS subject classifications: 65R20, 45E05

Key words: Convergence analysis, Legendre-spectral method, second order Volterra integro-differential equation, delay, error analysis.

1 Introduction

The paper is concerned with second order Volterra integro-differential equation with delay, which is as follow:

$$u^{(2)}(x)=\sum_{j=0}^{1}b_j(x)u^{(j)}(x)+\sum_{j=0}^{1}\int_0^{qx}k_j(x,s)u^{(j)}(s)ds+g(x),\quad x\in[0,T], \tag{1.1}$$

with

$$u(0)=u_0,\quad u^{(1)}(0)=u_1. \tag{1.2}$$

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Here q is a given constant with $0<q\leq 1$, and $b_j(x)$, $k_j(x,s)$, $g(x)$ are smooth functions on respectively domain. The vanishing delay is qx and $u(x)$ is an unknown function.

Since Volterra integro-differential equations with delay arise widely in the mathematical model of physical and biological phenomena, many researchers have developed theoretical and numerical analysis for the related types of equations. We refer the reader to [3, 6, 10] for a survey of early results on Volterra integro-differential equations. More recently, homotopy analysis method was used to solve system of Volterra integral equations (see, e.g., [14]) and polynomial spline collocation methods were investigated in [16, 19] and. In [20, 21], the authors used spectral collocation methods to study convergence analysis about Volterra integro-differential equations. For pantograph delay differential equations, in [2,12,22,24], the authors researched on these kinds of functions. In [1], spectral method was used to solve $y'(x)=a(x)y(qx)$, but it only analysed the numerical error in the infinity norm.

So far, very few work have touched the spectral approximation to second order Volterra integro-differential equations with delay. In practice, spectral method has excellent convergence property of exponential convergence rate. In this paper, we will provide a Legendre-collocation spectral method for the second order Volterra integro-differential equation with delay and analyse the numerical error decay exponentially in both L^2 and L^∞ space norms.

For ease of analysis, we will describe the spectral method on the standard interval $[-1,1]$. Hence, we employ the transformation

$$x=\frac{T}{2}(1+t),\quad t=\frac{2x}{T}-1.$$

Then the problem (1.1)-(1.2) becomes

$$y^{(2)}(t)=\sum_{j=0}^{1}B_j(t)y^{(j)}(t)+\sum_{j=0}^{1}\int_0^{\frac{qT}{2}(1+t)}\hat{k}_j(t,s)u^{(j)}(s)ds+G(t),\quad t\in[-1,1],\tag{1.3}$$

with

$$y(-1)=y_{-1}=u_0,\quad y^{(1)}(-1)=y_{-1}^{(1)}=\Big(\frac{T}{2}\Big)u_1,\tag{1.4}$$

where

$$\begin{aligned}
&y(t)=u\Big(\frac{T}{2}(1+t)\Big), && G(t)=\Big(\frac{T}{2}\Big)^2 g\Big(\frac{T}{2}(1+t)\Big),\\
&B_0(t)=\Big(\frac{T}{2}\Big)^2 b_0\Big(\frac{T}{2}(1+t)\Big), && B_1(t)=\Big(\frac{T}{2}\Big)b_1\Big(\frac{T}{2}(1+t)\Big),\\
&\hat{k}_0(t,s)=\Big(\frac{T}{2}\Big)^2 k_0\Big(\frac{T}{2}(1+t),s\Big), && \hat{k}_1(t,s)=\Big(\frac{T}{2}\Big)^2 k_1\Big(\frac{T}{2}(1+t),s\Big).
\end{aligned}$$

Furthermore we make a linear transformation: $s=T(1+\eta)/2$, $\eta\in[-1,t]$ and (1.3) becomes

$$y^{(2)}(t)=\sum_{j=0}^{1}B_j(t)y^{(j)}(t)+\sum_{j=0}^{1}\int_{-1}^{qt+q-1}K_j(t,\eta)y^{(j)}(\eta)d\eta+G(t),\quad t\in[-1,1],\tag{1.5}$$