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Modified Moving Least Square Collocation Method for Solving Wave Equations

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Abstract. This paper presents a modified moving least square (MMLS) collocation method for solving wave equations. In contrast to the conventional moving least square (CMLS) method, this method modifies how discretization of computational points is done and decreases the number of base functions to simplify shape functions while solving high-dimensional problems. In addition, the proposed method maintains the independence of discretization for different dimensions, which is convenient to deal with computational domains in a simple manner while retaining a local character. The above improvement results in this approximation significantly saving calculation time while preserving accuracy of the solution. The numerical simulations show that MMLS collocation method has good stability and accuracy in analyzing high-dimensional wave propagation.

AMS subject classifications: 65N80, 65N35, 65N38, 86-08

Key words: MMLS collocation method, wave propagation, base function.

1 Introduction

Wave propagation has extensive application in the field of engineering. In general, it is hard to obtain analytic solutions of wave equations, hence highly accurate numerical solutions play an important role in numerical solution of wave equations [1]. Nevertheless, the traditional grid method (finite element method, finite difference method, boundary element method) is limited by mesh generation in simulating high-dimensional wave propagation. To avoid these, the MMLS method exploits the advantages of the point collocated mesh-free method.

Over the past two decades, mesh-free methods [2, 3] have been presented to replace traditional grid methods and they have become one of the research hotspots in international computational mechanics. There are dozens of mesh-free methods at present and

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the major differences among the different mesh-free methods [4] are the construction of the shape function and the discrete version of the control equation [6,7]. The main methods to construct the shape function are the MLS methods [9], reproducing kernel approximation method [4, 5], the hp-cloud method [10], kernel function method and the point interpolation method [11, 12]. The principal methods of dealing with control equations are the collocation method [13] and the Galerkin method [2]. The core concept of the collocation method is that control equations should hold true for every node. Consequently, the collocation method is free from the mesh entanglement and mesh distortion, which may occur in the grid-based method. The obvious advantages of collocation method are high computational efficiency and easy achievement of displacement boundary conditions.

This proposed approximation is based on moving least square technique [14, 15] and a simple point collocation procedure [16, 17]. Improving the discretization of computational area and decreasing the base functions within high-dimensional problems are the main contributions of MMLS method. Using the above modification, there is no difficulty in refining the problem in the area of interest; one needs only to add points within the interest area as well as simplify the shape function to greatly enhance calculation efficiency.

In this paper, the principle of MMLS method is described in the first part. In the next part, the comparison between MMLS and CMLS suggests that MMLS possesses a greater advantage in computing efficiency and accuracy. An example follows regarding 2-dimensional wave propagation, which illustrates how MMLS preserves considerable precision with different-scale discretization. Finally, the reliability of MMLS within high-dimensional problems is demonstrated through the numerical simulation of 3-dimensional wave propagation.

2 Derivation of MMLS method within wave propagation

2.1 Moving least square interpolation

In general, moving least square (MLS) is considered to be one of the best methods to approximate random data with a reasonable precision due to its completeness, robustness and continuity. MLS is introduced in MMLS formulation to construct shape functions via discrete points. For simplicity, the 1D implementation of MLS is reviewed below:

With MLS approximation, a function $u(x)$ is approximated over a series of discrete nodes in interested region

$$u(x)=\sum_{i=1}^{m}N_i(x)a_i(x)=p^T(x)a(x), \tag{2.1}$$

where the linear basis polynomial is $p^T(x)=[1,x]$, and the quadratic basis polynomial is $p^T(x)=[1,x,x^2]$ for one-dimensional problems. The coefficient $a(x)$ is determined by