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On the Comparison Between Lattice Boltzmann Methods and Spectral Methods for DNS of Incompressible Turbulent Channel Flows on Small Domain Size

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Abstract. The paper presents a study of the influence of the domain size and LBM collision models on fully developed turbulent channel flows. The results using spectral method show that a smaller domain size will increase the velocity fluctuations in the streamwise direction. And MRT-LBM with different collision models gives reliable results at least for low order flow statistics compared with those from spectral method and finite-difference method.

AMS subject classifications: 76F65, 76M28

Key words: Lattice Boltzmann method, Spectral method, turbulent channel flows.

1 Introduction

Over the last 30 years, direct numerical simulations (DNS) have been applied to investigate a large number of turbulent flows, which generates more details about the mechanisms of turbulent transport than experimental methods. For simple geometries, many algorithms have been applied to simulate turbulence, e.g., finite-difference method, pseudo-spectral method and Lattice Boltzmann Method (LBM). Pseudo-spectral method is considered to be the most reliable method for its high-order spatial accuracy and low numerical dissipation when compared at the same temporal and spatial resolutions

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with alternative methods. However, when the geometries are complex, finite-difference method and pseudo-spectral method are difficult to be used. A fully mesoscopic LBM as an alternative approach to incompressible turbulence is easy to deal with complex geometries and easy to parallelize. This offers the potential for the LBM to be applied to treat turbulent particle-laden flows.

Fully developed incompressible turbulent channel flows are considered here. The benchmark Navier-Stokes solutions simulated by Chebyshev pseudo-spectral method KMM [1] are used as a comparison. Lammers et al. [2] reviewed the former studies of LBM DNS on developed channel flows and also used a BGK (single-relaxation-time) D3Q19 model to simulate the channel flow turbulence which is in great agreement with the result of KMM. Suga [3] used a MRT(multiple-relaxation-time) D3Q27 model and also obtained good results. Freitas [4] performed a comparative study of different discretizations, i.e., D3Q19 and D3Q27 model, applied to a turbulent channel flow at $Re_\tau = 200$. Averaged streamwise velocity and rms(root-mean-squared) velocity are reasonable for both D3Q19 and D3Q27. However, the peak of the streamwise component of the Reynolds stress is more accurately predicted by the D3Q27 model. Kang and Hassan [5] reported that the D3Q19 model breaks the rotational invariance and produces unphysical results especially for turbulent wall-bounded flows–turbulent circular pipe and square duct flows, while the D3Q27 model achieves the rotational invariance. Thus, we use the D3Q27 model in this paper.

Gehrke et al. [6] reported the potential influence of the collision models in LBM, i.e., the BGK, the MRT and the Cumulant model on DNS of turbulent channel flows. The BGK model is the single-relaxation-time model, which is simple and often become unstable when the Reynolds number is high. Then multiple-relaxation-time (MRT) model is proposed by d'Humieres [7] to overcome this difficulties. MRT model allows each physical process to have its own relaxation time in order to improve the numerical stability and physical accuracy. The results show that the mean velocity profiles of the MRT and the Cumulant model are slightly higher than the BGK model and KMM. Wang et al. [8] used LBM and Yu et al. [9] used finite-difference method to study a smaller domain size ($4H \times 2H \times 2H$) turbulent channel flows. Yu also obtained a higher mean velocity profile and they both found a higher rms velocity in the streamwise direction. We will use the pseudo-spectral method to show the influence of the domain size in Section 3.1 and compare the different LBM-MRT D3Q27 models on small domain size ($4H \times 2H \times 2H$) in Section 3.2.

2 Numerical methods

2.1 Lattice Boltzmann method

We use the MRT-LBE collision model with Q discrete velocities in three dimensions. The evolution equation for the MRT-LBE on each lattice node $\mathbf{x}$ at time t can be written as the