Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **11**, No. 3, pp. 630-639

DOI: 10.4208/aamm.2018.s07
June 2019

Acoustic Mode in Plane Free Mixing Layers

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Received 25 August 2018; Accepted (in revised version) 6 December 2018

Abstract. The linear stability of supersoinc mixing layer with the mean velocity profile approximated by the hyperbolic tangent is studied. The convective Mach number of mixing layer is 2 and the temperature ratio is 1. A new classification of disturbance mode of mixing layer is presented to unify the existing results in literature and the new results in this paper. The mixing layer has supersonic and subsonic mode according the disturbance phase speed. Then the supersonic mode can be classified into mixing mode and asouctic mode. The mixing mode can be further classified into sbsonic-supersonic mode and supersonic-subsonic mode according the relative speed between disturbance phase speed and the free flow speed of mixing layer, which correspond to fast mode and slow mode, respectively. The pahse speed of acoustic mode are supersonic with respet to the free flow in both sides of mixing layer and some unstable acoustic mode are found. The acoustic mode has radiation characteristics in both sides of mixing layer and results in a series of expansion and compression fans.

AMS subject classifications: 65M10, 78A48

Key words: Supersonic mixing layer, linear stability, disturbance mode.

1 Introduction

The mixing layer that forms between two fluid streams moving with different velocities is an important model problem for the study of turbulence and is also a prototype flow for the study of mixing process involved in such chemically reacting systems of the scramjet engine for the propulsion of hypersonic aircraft. During the past four decades, many studies have made major advances in the understanding of compressible mixing layer. However, many questions remain unanswered, especially for the situation that the convective Mach number $M_c > 1$. Convective Mach number M_c is the compressibility parameter of mixing layer, which was introduced by Bogdanoff [1] and Papamoschou & Roshko [2]. It was defined by

$$M_c = \frac{U_1 - U_2}{a_1 + a_2}, \tag{1.1}$$

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where U_1 and U_2 are the free-stream velocities, a_1 and a_2 are the free-steam sound speed. In this work we focus on the stability characteristics of compressible mixing layer at high convective Mach number, which is of fundamental interest and is also important in engineering application. This is because these stability characteristics may allow one, in principle, to control the downstream evolution of such flows [3].

There have been a number of theoretical studies [4–7] on the stability of compressible mixing layer. In 1989, Jackson and Grosch [8] reported their results about the stability of compressible mixing layer over a large Mach number range $0\leq M_1\leq 10$ while M_2=0. A complete view of the inviscid stability of compressible mixing layer with the mean velocity profile approximated by the hyperbolic tangent has been shown. They pointed out that there are four disturbance modes, including subsonic modes, fast modes, slow modes and supersonic-supersonic modes. The subsonic modes are both subsonic with respect to the two free-streams. On the contrary, the supersonic-supersonic modes are both supersonic with respect to the two free-streams. Besides this, the fast modes are subsonic with respect to the moving stream and supersonic with respect to the stationary stream. Then the slow modes are supersonic with respect to the moving stream and subsonic with respect to the stationary stream. Jackson and Grosch [8] found some unstable subsonic, fast and slow modes but did not find any neutral or unstable supersonic-supersonic modes. They also noted that both the fast and slow supersonic modes are vortical modes and neither of them is an acoustic mode. Vortical modes correspond to the Kelvin-Helmholtz instability [9] of incompressible mixing layer, which is the dominant mechanisms of the roller and rib structures of mixing layer. And acoustic modes have radiation characteristics in the far field [10]. The fast and slow modes are also found by Liang & Reshotko [11] and Zhuang & Dimotakis [12]. In 2007, Shen et al. [13] studied the supersonic mixing layer at $M_c=1.2$ by using linear stability theory and direct numerical simulation methods, and then pointed out that both the fast and slow supersonic modes are acoustic radiation vortical modes.

Therefore, it can be found that the definition of disturbance modes in free mixing layers are not very clear. Furthermore, the unstable supersonic-supersonic modes are not found until now. Here linear stability analysis of a plane free mixing layer at $M_c=2$ is carried out to study these issues. There are two obvious differences from the job of Jackson and Grosch [8]. One is that the two free streams are moving with supersonic speed in the present study while one of the two streams is stationary in the job of Jackson and Grosch [8]. Another is that the stability analysis here is viscosity while the stability analysis in the job of Jackson and Grosch [8] is inviscid.

2 Linear stability analysis

The dimensionless parameters of supersonic mixing layers are shown in Table 1, where the characteristic speed is the sound speed of high-speed flow and equals to 197.15m/s, the characteristic temperature is the temperature of high-speed flow and equals to 100.0K,