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Posteriori Error Estimation for an Interior Penalty Discontinuous Galerkin Method for Maxwell's Equations in Cold Plasma

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Abstract. In this paper, we develop a residual-based a posteriori error estimator for the time-dependent Maxwell's equations in the cold plasma. Here we consider a semi-discrete interior penalty discontinuous Galerkin (DG) method for solving the governing equations. We provide both the upper bound and lower bound analysis for the error estimator. This is the first posteriori error analysis carried out for the Maxwell's equations in dispersive media.

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Key words: Posteriori error estimator, Maxwell's equations, cold plasma, discontinuous Galerkin method.

1 Introduction

Dispersive electromagnetic media are those materials with wavelength dependent physical parameters (such as permittivity). Examples of dispersive media include human tissue, soil, snow, ice, plasma, optical fibers and radar-absorbing materials. Hence, the study of wave interaction with dispersive media is very important to our daily life.

In recent years, there is a growing interest in the finite element modeling and analysis of Maxwell's equations (see books [10, 17, 30] and references cited therein). However, most work is restricted to the discussion of simple medium such as air in the free

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space. Work on dispersive media is still very limited. In 2001, Jiao and Jin [22] initiated the application of time-domain finite element method (TDFEM) for the dispersive media. Then in 2004, Lu et al. [28] developed a discontinuous Galerkin (DG) method for solving Maxwell's equations in dispersive media. In 2005, Banks and Browning [5] considered a Debye medium problem solved by finite element method. Unfortunately, no any error analysis has been carried out for Maxwell's equations in dispersive media. Since 2006, we carried out some a priori error analysis of TDFEM for dispersive media [21,23–27]. In this paper, we initiate our effort on developing a posteriori error estimation for Maxwell's equations in dispersive media. For simplicity, we only consider the cold plasma model in this paper. Analysis of other dispersive media can be carried out similarly.

A posteriori error estimation plays an important role in adaptive finite element methods (FEMs), and the literature on this is vast (see books [1,3,4,34], reviews [11,13] and references cited therein). However, to our best knowledge, there are only dozens of papers devoted to the study of posteriori error estimation for Maxwell's equations [6–9,16,19,20,29,32,35]. No any paper has discussed the posteriori error estimation for dispersive media yet. Here we want to fill the gap by carrying out the first posteriori error analysis for the Maxwell's equations in dispersive media.

The governing equations that describe electromagnetic wave propagation in isotropic nonmagnetized cold electron plasma are [25]

$$\epsilon_0 \boldsymbol{E}_{tt} + \nabla \times (\mu_0^{-1} \nabla \times \boldsymbol{E}) + \epsilon_0 \omega_p^2 \boldsymbol{E} - \nu \boldsymbol{J}(\boldsymbol{E}) = 0, \tag{1.1}$$

where $\boldsymbol{E}$ is the electric field, ϵ_0 is the permittivity of free space, μ_0 is the permeability of free space, ω_p is the plasma frequency, $\nu \geq 0$ is the electron-neutral collision frequency, and the polarization current density $\boldsymbol{J}$ is represented as

$$\boldsymbol{J}(\boldsymbol{x}, t; \boldsymbol{E}) \equiv \boldsymbol{J}(\boldsymbol{E}) = \epsilon_0 \omega_p^2 \int_0^t e^{-\nu(t-s)} \boldsymbol{E}(\boldsymbol{x}, s) ds. \tag{1.2}$$

Moreover, we assume that the boundary of Ω is a perfect conductor so that

$$\boldsymbol{n} \times \boldsymbol{E} = 0 \quad \text{on} \quad \partial\Omega \times (0, T), \tag{1.3}$$

where $\boldsymbol{n}$ denotes the unit outward normal of $\partial\Omega$. Furthermore, we assume that the initial conditions for (1.1) are given as

$$\boldsymbol{E}(\boldsymbol{x}, 0) = \boldsymbol{E}_0(\boldsymbol{x}) \quad \text{and} \quad \boldsymbol{E}_t(\boldsymbol{x}, 0) = \boldsymbol{E}_1(\boldsymbol{x}), \tag{1.4}$$

where $\boldsymbol{E}_0(\boldsymbol{x})$ and $\boldsymbol{E}_1(\boldsymbol{x})$ are some given functions.

The rest of the paper is organized as follows. In Section 2, we describe the semi-discrete DG formulation for the plasma model. In Section 3, we construct our a posteriori error estimator and present the main result. Detailed proof of the error estimator is given in Section 3.1. Here we adopted many ideas and techniques from [20] originally developed for Maxwell's equations in the simple medium. Then in Section 4, we prove some lower bounds for the local error estimators. We conclude the paper in Section 5. In this paper, C (sometimes with subindex) denotes a generic constant which is independent of both the time step τ and the finite element mesh size h.