Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **1**, No. 1, pp. 1-28 (2009) 

# Detection of Arterial Occlusions Using Viscoelastic Wave Propagation

H.T. Banks[1,*] and J.R. Samuels, Jr.[1]

[1] *Center for Research in Scientific Computation, North Carolina State University, Campus Box 8212, Raleigh, NC 27695-8212, USA.*

Received 13 December 2008; Accepted (in revised version) 4 February 2009

Available online 16 February 2009

---

**Abstract.** We consider the problem of detecting cardiac artery occlusions using stenosis driven viscoelastic (VE) waves propagated through biotissue to body surface sensors. We investigate possible statistical model formulations (ordinary least squares (OLS), generalized least squares (GLS)) and post analysis techniques (residual plots) to ascertain uncertainty in estimates as well as validity of the statistical models as part of a methodology for stenosis detection using viscoelastic waves.

**AMS subject classifications**: 34A55, 35R30, 74J25, 62J02, 93A30

**Key words**: Inverse problems, viscoelastic models, wave propagation in biotissue, statistical models.

---

## 1 Introduction

Coronary artery disease (CAD) is the leading cause of death in many parts of the developed world. The disease is caused by a gradual accumulation of atheromatous plaques (e.g. cholesterol, fatty acids, calcium and fibrous connective tissue) along the wall of the vessel [1]. Although the symptoms of CAD are pronounced in the later stages of the affliction, it is very difficult to diagnose the disease before the first onset of symptoms, typically a sudden heart attack [24].

Current detection techniques include angiograms and CT scans. Angiograms are an invasive technique, which typically requires entering the vessel through the femoral artery or the jugular vein — a process many may wish to avoid. On the other hand, CT scans generate a three-dimensional image of an object by taking multiple two-dimensional X-Rays while rotating around an axis and expose a patient to sufficient radiation. In another approach, engineers have developed sensors that can detect the

*Corresponding author.
*URL:* `http://www.ncsu.edu/crsc/members/banks.html`
*Email:* `htbanks@ncsu.edu` (H.T. Banks), `samuels.johnny@gmail.com` (J.R. Samuels, Jr.)

 

acceleration of acoustic waves [2]. The underlying approach is rather simple: blockages in the artery create turbulence in the blood flow, which then generates an acoustic wave with a normal and shear component (Fig. 1). The acoustic wave propagates through the chest cavity until it reaches the chest wall, where a series of sensors would detect the acceleration of the components of the wave. The data from the sensors might then be used to quickly determine the location of the blockages, if any, in the artery. Unlike the other detection techniques discussed above, the process would be inexpensive and non-invasive.

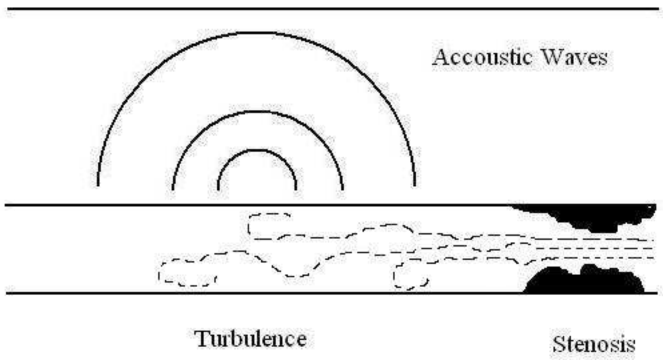


Figure 1: Depiction of interaction between arterial blockage and acoustic wave.

For such a technology to be feasible, a mathematical model that describes the propagation of the acoustic wave from the stenosis to the chest wall would be necessary. One, two and three dimensional models have been developed [2, 23, 25] as investigators continued efforts on scientific issues, especially the feasibility of detecting and locating the stenosis. In [11, 23], the authors identified difficulties with obtaining estimates of the stenosis parameters as well as reliability of these and other estimates of parameters even in the two-dimensional models. Here we report further efforts and results related to the efficacy of the inverse problem involving a two-dimensional version of the model and simulated noisy data.

In particular we consider viscoelastic (VE) models for stenosis driven wave simulations and demonstrate that the corresponding inverse problems are computationally tractable even though they involve computationally challenging forward problems. Specifically, we show that one can implement the inverse problem for estimation of both geometric and material parameters in the VE wave propagation models. We consider how one ascertains reliability of estimates and show by example one approach to the computation of standard errors and the associated confidence intervals [5,14,16,26]. As we detail below, there exist asymptotic theories for error analysis – whether for absolute error or for relative error in the data, but one does not know *a priori* what type of error model to assume. As we demonstrate by computational examples, one must use the correct statistical model for the standard errors computed to be meaningful (one could err because one could assume incorrect error $\epsilon$ or incor-